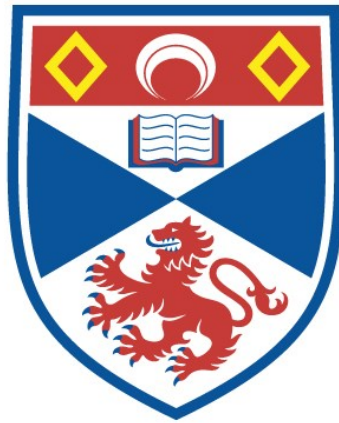


PROPER NAMES AND POSSIBLE WORLDS

Roderic Allen Girle

A Thesis Submitted for the Degree of PhD
at the
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ABSTRACT

In this essay a theory of proper names is developed and applied to the construction of quantified modal logics and to a discussion of problems concerning identity across possible worlds. The theory is then used to aid discussion of essentialism, empty singular terms, quantification into epistemic contexts, and Frege's problem with identity.

In the first chapter, after a preliminary discussion of Russell's and Frege's theories of names, a theory is developed. It is argued that in the giving of a name a relation is established between the name and what is named. That relation is the sense of the name. It is also argued that names can be given to imaginary, fictional, and other such non-existent things.

The second chapter is devoted to a discussion of Quine's programme for eliminating singular terms. It is there argued that the programme cannot be justified.

The third chapter centres around the construction of logical systems to deal with identity across possible worlds. It is assumed that once a name is given and its sense thereby established the name is a rigid designator. Quantificational systems are constructed without modal operators yet in terms of which cross world identity can be discussed. Modal operators are then introduced to facilitate a discussion of essentialism and identity. At each point the formal systems are constructed in accordance with clearly stated assumptions about constant singular terms, the domains of quantification, and the interpretation of modal operators.

The fourth chapter is a discussion of essentialism. There is a discussion of radical change from world to world and of giving accounts of such changes, and a discussion of the idea that primacy should be given to the sense which a name has in one particular world.

The fifth chapter is a discussion of empty singular terms and free logics. There is a discussion of the nature of the values of empty singular terms in the formal semantics of free logics, and a discussion of the objectual and substitutionary interpretations of quantifiers.

The final chapter is a discussion of quantification and identity in epistemic logic. It is argued that the theory of proper names developed is of use in clarifying questions about quantification into epistemic contexts and about Frege's problem with the cognitive values of "a = a" and "a = b" when both are true.

Roderic Allen GIRLE

TO

The University of St. Andrews

A Thesis for
the Degree of Doctor of Philosophy

'Proper Names and Possible Worlds'

by

Roderic Allen GIRLE



Th 8643

I, Roderic Allen GIRLE, hereby declare that this thesis is my own work, that I am the sole author, that the work of which it is a record is my own, and that this work has not been accepted in any previous application for a higher degree.

Signed:

I was admitted as a Research Student of the University of St Andrews in October 1970 in the Faculty of Arts for the degree of Doctor of Philosophy. My Supervisor has been Mr. G.B.B. Hunter of the Department of Logic and Metaphysics. For the first three terms I worked as a full-time student, and for the other twelve terms I have worked as a part-time external student.

Signed:

I hereby certify that the conditions of the Resolution and Regulations for the Degree of Doctor of Philosophy of the University of St Andrews have been fulfilled in the case of Mr Roderic Allen Girle.

Signed

(Supervisor)

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INTRODUCTION

Fictional characters, animals or places are often given names. Those names, such as "Pickwick", "Pegasus" or "CairParavel", are then used when we talk about such fictions. A philosophical consideration of the use of such names has raised the question, How can we talk about something which does not exist, since there is nothing about which to talk?

At first sight this question may strike us as ambiguous because of a kind of play upon the words "something" and "nothing". So let us consider briefly the way in which Russell came to the conclusion that the names of fictions, and probably most other names also, are disguised descriptions. The way in which Russell proceeded will give point to the question posed above.

"A name," says Russell, "is a simple symbol, directly designating an individual which is its meaning, and having this meaning in its own right, independently of the meanings of all other words."¹ For Russell, the bearer of a name is the meaning of that name. Russell also assumes that in a genuine subject-predicate proposition² the meaning of the subject term is whatever is designated by this term. Under Russell's assumptions the assertion "Socrates is wise" can be analysed as saying of the bearer of the name 'Socrates' that he has the characteristic of being wise. But if the bearer does not exist then the subject term is meaningless and the assertion meaningless. Only if the bearer exists can the assertion

¹B. Russell, Introduction to Mathematical Philosophy. Allen and Unwin, London, 1920. P. 173.

²As opposed to the Aristotelian so called "subject-predicate proposition".

be either true or false. How then could we assert meaningfully that Excalibur was brandished, or that the round square does not exist? The question is even more acute in the case of the round square because we can hardly be asserting that there is an object which is not.

Russell holds that the meaning of a proper name is its bearer. This is a key assumption. If there is no bearer existent, as in the case of fictional characters, then the name is meaningless, and the assertions in which such names occur are meaningless. Since we cannot agree that assertions about fictional characters and such like are meaningless it follows that either Russell was mistaken about proper names or, as he was to suggest, most of the terms which we call proper names are not proper names at all. Most of what we call proper names are really disguised descriptions, says Russell, so that

whenever the grammatical subject of a proposition can be supposed not to exist without rendering the proposition meaningless, it is plain that the grammatical subject is not a proper name, i.e., not a name directly representing some object. Thus in all such cases, the proposition must be capable of being so analysed that what was the grammatical subject shall have disappeared.¹

Russell then proposed analyses in terms of definite descriptions.

Our concern in this essay is with proper names. We will not be much concerned with the theory of descriptions as such. Our main concern will be to consider the status of names in language and what can be said about their meaning.

We begin to develop a theory of proper names by discussing Russell's assumptions and Frege's theories. In the course of discussion we will consider Russell's idea that if a sentence is logically of the subject-predicate form, "then the very fact of its being

¹B. Russell. Principia Mathematica Vol. 1, 2nd Ed. Cambridge University Press, 1925, p. 66.

significant, having a meaning guarantees that there is something referred to by the logical (and grammatical) subject."¹ A theory of proper names will then be proposed, a theory somewhat in the style of Frege's theory with emphasis on the sense of names.

Even so, a theory of proper names would be somewhat redundant if it were possible to purge a language of all names. So we discuss Quine's programme for the elimination of singular terms.

The theory of proper names will then be applied to discussions of several questions of current interest in the philosophy of logic. First we discuss the problems of identity across possible worlds, then essentialism, free logic and quantification into epistemic contexts. At the conclusion of the last discussion the theory is applied to one of Frege's problems about identity.

¹L. Linsky, Referring. Routledge and Kegan Paul, London, 1967, p. 67.

CHAPTER ONE

THE MEANING AND REFERENCE OF NAMES

Russell on names

Russell's two assumptions about proper names were that (a) the meaning of a proper name is its bearer, and (b) the only bearers of proper names to be countenanced are real or existing objects. We consider first the notion that the meaning of a proper name is its bearer.

One is tempted to say straight away that Russell is plainly mistaken when he says that proper names have any meaning at all. If one were asked what was the meaning of some proper name, such as "Robin", one would be likely to reply either that names don't have meanings, or to reply by asking the questioner what the point of his question was. Of course, the questioner might be asking the kind of etymological question which can be given an answer such as "'Robin' means 'courageous and just'", or "'Emmanuel' means 'God with us,'" There is also the weak sense in which "Bill" means "male", but then "Bill" has been used as the short name for women named "Wilhelmina".

The temptation to say that Russell is simply mistaken can be resisted if we take into account some comments which he makes about the use of the term "meaning". In his lectures on Logical Atomism he says

As to what one means by 'meaning', I will give a few illustrations. For instance, the word 'Socrates', you will say, means a certain man; the word 'mortal' means a certain quality; and the sentence 'Socrates is mortal' means a certain fact. But these three sorts of meaning are entirely distinct, and you will get into the most hopeless contradictions if you think the word 'meaning' has the same meaning in each of these three cases. It is very important not to suppose that there is just one thing which is meant by 'meaning', and that therefore there is just one sort of relation of the symbol to what is

symbolized in the case of a name, there is only one relation that it can have to what it names. A name can just name a particular, or, if it does not, it is not a name at all, it is a noise.¹

So it's reasonable to say that Russell's account of proper names is that they simply designate individuals. There are other points in his account of proper names, but they need not concern us just now because they bear chiefly on the status of what is designated, and the relationship between the user of a proper name and what it designates. We can usefully label Russell's account of proper names as a Simple Referential theory. The term "Simple" is used in order to highlight a contrast which can be drawn between Russell's account of proper names and Frege's account.

Theories of meaning

Before going on to consider Frege's account of proper names we need to be clear about what the larger story is into which these various accounts of proper names fit. The larger story will be some theory of meaning. It will suffice, for our purposes, to adopt what Dummett says about theories of meaning in his study of Frege.² Although what Dummett says is specific to Frege's work, we can generalise from what he says without prejudicing any contrasts we may want to highlight, in this essay, between Frege and other philosophers such as Russell.

What Dummett says can be put as follows. First, for any theory of meaning the sentence is the unit of meaning. Secondly,

¹"The Philosophy of Logical Atomism" in Logic and Knowledge, Ed. R.C. Marsh. Allen & Unwin, London, 1956, pp.186-187.

²Michael Dummett, Frege's Philosophy of Language. Duckworth, London, 1973.

a theory of meaning must be based on an analysis which does "not stop short at the specification of which sentences are well formed: it must explain also how the meaning of each sentence (is) determined from its internal structure . . . it must be a semantic, and not merely a syntactic, analysis."¹ Thirdly, a distinction can be drawn between the sense and the tone of a sentence or expression: "to the sense of a sentence belongs only that which is relevant to determining its truth or falsity; any feature of its meaning which cannot effect its truth or falsity belongs to its tone. Likewise, to the sense of an expression belongs only that which may be relevant to the truth or falsity of a sentence in which it might occur; any element of its meaning not so relevant as part of its tone."² An example of expressions with the same sense but different tone is the pair: "and" and "but".

Russell's account of proper names can be seen as saying that the sense of a name consists just in its having a certain referent. But this account does not satisfy Frege.

Frege and identity

If proper names simply designate individuals and have no other vital characteristic, then there is a problem which arises. The problem concerns identity-propositions such as "Cicero is Tully" and "The morning star is the evening star." About identity Frege asks, "Is it a relation? A relation between objects, or between names or signs of objects?"³ The first alternative, that identity is a

¹Ibid., p. 2.

²Ibid., p. 2.

³G. Frege, "On Sense and Reference" in Translations from the Philosophical Writings of Gottlob Frege, Eds. P. Geach and M. Black. Basil Blackwell, Oxford, 1960, p. 56.

relation between objects, is very much in accord with what Russell says about assertions containing proper names. Since "Socrates is wise" can be analysed as saying of the bearer of the name "Socrates" that he has the characteristic of being wise, "Cicero is Tully" should be analysed as saying of the bearers of the names "Cicero" and "Tully" that they have the characteristic of being identical with each other.

Taking the first alternative Frege says,

Now if we were to regard equality as a relation between that which the names 'a' and 'b' designate, it would seem that $a=b$ could not differ from $a=a$ (i.e. provided $a=b$ is true).¹

But " $a=a$ " is different from " $a=b$ ". The difference that concerns Frege here is not simply a syntactic difference but a difference in terms of information. " $a=a$ " conveys quite different information from the information conveyed by " $a=b$ ". "The two sentences," says Frege, "do not have the same cognitive value",² and this is just what is meant by saying that the information conveyed by one is different from the information conveyed by the other.

As Dummett says,

The notion of 'information' being appealed to here does not require any elaborate explication: I acquire information when I learn something which I did not previously know, and Frege is asking how it is possible that I may be in a position to know the sense of an identity-statement, i.e. to understand it, and yet learn something that I did not know before by being told that that statement is true.³

It is important to realize that implicit in Frege's criticism of a simple referential account of the sense of 'a' and 'b' in " $a=b$ " is the notion that "a theory of meaning is a theory of

¹Ibid. p. 56.

²Ibid. p. 78.

³Op.cit. p. 95.

understanding. What we have to give an account of is what a person knows when he knows what a word or expression means, that is, when he understands it."¹ Now, if when a person understands 'a's being a name he knows its referent, and similarly for 'b', then that person will know that $a=b$. "If the sense of a name consists merely in its reference, anyone who understands two names having the same referent must know that they have the same referent."²

Although this looks like a most powerful criticism of Russell's account of proper names, Russell has a ready reply. The reply is contained in the following statements of Russell's:

For a logically perfect language, there will be one word and no more for every simple object,...³

...all the names that it would use would be private to that speaker and could not enter into the language of another speaker.⁴

Russell really does meet Frege head-on here. Real proper names would be able to be used only by people who know the referent of the name, and identity statements would be non-existent because they would be useless. It is just because Russell conceives of an ideal language in this way that his account is unrealistic, but it is consistent. We will return again to this question of the user of proper name having to be acquainted with the name's referent.

Taking the other alternative, that is that identity is a relation between names, Frege argues that " $a=b$ " is just not an assertion about words. If we take " $a=b$ " to say that the name 'a'

¹Ibid. p. 92.

²Ibid. p. 95.

³Op.cit., p. 197.

⁴Ibid. p. 198.

and the name 'a' are names for the same thing then, according to Frege, we would be saying no more than that we had arbitrarily agreed to use these words as names for the same object, whatever that object might be. "In that case the sentence $a=b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means. But in many cases this is just what we want to do."¹

Frege then presses his argument even further and says that we cannot, anyway, look upon names simply as objects of some sort, for example, typographical objects. We have to take account both of what a name refers to (its reference) and the way in which a name designates an object (its sense). It is not altogether clear just what Frege is talking about when he talks about the mode of designation.

He says:

If the sign 'a' is distinguished from the sign 'b' only as object (here, by means of its shape), not as sign (i.e. not by the manner in which it designates something). The cognitive value of $a=a$ becomes essentially equal to that of $a=b$, provided $a=b$ is true. A difference can arise only if the difference between the signs corresponds to a difference in the mode of presentation of that which is designated.²

One way of seeing Frege's argument and point here is to see him as saying that names cannot be distinguished one from the other simply by means of syntax. 'a' and 'b' are not different names simply because they are two different items on a list of primitive symbols. So if they are different names their difference must lie in their having different relationships with the one thing to which both refer. As Dummett explains:

¹Op.cit. p. 57

²Ibid. p. 57

Hence two names may have the same referent but different senses: with the two names are associated different methods of identifying some object as the referent of either name, although it happens that it is the same object which satisfies the two pairs of conditions of such identification.¹

Frege then explains that,

The regular connection between a sign, its sense, and its reference is of such a kind that to the sign there corresponds a definite sense and to that in turn a definite reference, while to a given reference (or object) there does not belong only a single sign.²

In this way the sense of a proper name has its place in determining the truth-value of any sentence in which the proper name occurs.

"The sense of a word -- as opposed to any other ingredient its meaning may have -- constitutes the contribution which it makes to determining the truth-condition of sentences in which it occurs precisely by associating a certain reference with it."³ Yet, even though the sense of a name associates a certain reference with it, for each name there will be a different way of associating its referent with it, and because of the differences between these ways of association the names will be different.

Names and meaning

From what Frege says about identity taken as a relation between names it is not altogether clear that he is not just wrong. He expounds this alternative, as we have seen, as "What is intended to be said by $a=b$ seems to be that the signs or names ' a ' and ' b ' designate the same thing, so that those signs themselves would be

¹Op.cit. p. 95

²Op.cit. p. 58

³Dummett, op.cit. p. 93.

under discussion; a relation between them would be asserted."¹ His criticisms of this approach are (a) that it makes names quite arbitrary, and (b) that it makes $a=b$ no longer "refer to the subject matter, but only to its mode of designation."²

It is not altogether clear just what Frege's first criticism amounts to, but it is hard to see how one can avoid the conclusion that names are arbitrarily chosen and given to people, places and things, etc. It may be that this criticism springs partly from Frege's inclusion of definite descriptions, such as "the morning star", in the general category of proper names. But even if this criticism does spring partly from the way in which definite descriptions are categorised it is hard to see how the criticism survives Frege's own example of a mountain with the two names 'Afla' and 'Ateb'.³ These two names are arbitrarily chosen, even made-up, words. Once chosen and declared to be names then the words can be used by any speaker of the language to refer to some particular. Words can be declared to be names by any one of several ways such as by being used in a rite such as baptism, or by some declaration of an overt kind like "The man, whose name was 'Brinklethorpe', . . .", or just by being used in literature with a capital first letter. Although the choice of some word as the proper name of some particular may be quite arbitrary, that is not to say that there is a subsequent randomness about the use of such words or the category to which they are seen to belong. "Peter", "Rover" and "Ateb" have a quite firm status as names, and as three different names.

¹Op.cit. p. 56

²Ibid. p. 57

³See Dummett, op.cit. p. 97.

And while we are discussing the syntactic category of names, and distinguishing between how a word comes to be included in the category and what status the word then has in a language, we need to note also that a name is not merely a phonological or typographical object, token or even type.

In most languages a is the same name as a, and b is a different name. In most languages we tell that a name a is the same name as the name a because the typographical tokens are both of the same typographical type. Similarly, we distinguish a from b typographically. This practice can be the source of a confusion. We could confuse the nature of the similarities and differences between typographical marks (or spoken words) with the nature of the similarities and differences between names.

There are the sorts of languages suggested by Searle¹ in which there is some convention for changing the typographical or phonological object in order as we use a name. For example, the first time we use some given name we use the mark "a", the second time the mark "b", the third time the mark "c", and so on.

So, in this sense, Frege is correct when he says that we cannot look upon names simply as typographical objects, or objects of some other similar sort. But, although we cannot look upon names as objects of that kind, it does not follow that we cannot take "a=b" to be saying something about names in their role in a language as names. Hence, if we take it at the outset that it is names that we are talking about, not typographical objects, then "a=b" can be about names as such, and not be about typographical objects.

Frege himself uses the term "sign" when talking about

¹J.R. Searle "Proper Names". Mind, 67, April 1958. p. 167.

names. We could say that there is a class of signs which is the class of names. These names are to be distinguished syntactically. They are, in fact, a set of syntactic objects. But then, it might be objected that we are still left with the problem of how "John" and "James" are really two distinct elements in the set of names. Our arguments to show that one name can be expressed typographically (or phonologically) in several different ways will only give added force to the objection that perhaps "John" (in the typographical sense) is expressive of the same name as "James" in the typographical sense. Frege would say that they are different because their senses are different. If there is no need to provide other than syntactic criteria for sameness of names then Frege's bringing in of the senses of names is unnecessary. If there is a need to provide other than syntactic criteria for sameness of names then Frege's bringing in of the senses of names is dangerous to his own account. It is dangerous because there is not usually some one way of associating the referent of a name with the name which is understood by all those who use the name meaningfully.

This last point is not that the notion of a name's having sense and reference is somehow self-defeating. Provided we accept that there are names, and that the syntax of the language indicates which names are which, then the notion of these names' having both sense and reference is not unreasonable, and it is also not unreasonable to agree that the sense of a name may vary from speaker to speaker. "Frege . . . was perfectly well aware of the variations in sense attached by individuals or at different times to the same expression, and of the haziness of the senses so attached."¹ The point at issue is

¹Dummett, op.cit. p.103.

that we cannot use either sense or reference as criteria for identity of names themselves. Here we have to rely on syntactic criteria.

We now come to Frege's second criticism. His criticism is that if we say that "a=b" says that what is named by 'a' is also named by 'b', then "a=b" no longer refers to the subject matter but only to its mode of designation. From this it would follow that no real information would be imparted about the referents of 'a' and 'b'.

If there is an argument here it is certainly invalid, but more importantly, it is just not true that if we say that "a=b" says that what is named by 'a' is also named by 'b', then "a=b" no longer refers to the subject matter but only to its mode of designation. Apart from the fact that it is strange to hear Frege talking about what "a=b" is referring to in terms of particulars, there is a formalization of "a=b" which is illuminating. If we let "Nxy" mean "x names y", then we have the following formalization of the view criticised:

$$a = b =_{df} N'a' (\exists x) N'b'x$$

The definiendum

can be read as

"The item which 'b' names is named by 'a'."

In this case the subject matter, i.e. the item named, is most certainly discussed and the information imparted is not equivalent to a logical truth. There is proper knowledge about the referent in "a=b" of 'a' and of 'b'.

Although Frege's criticism of the simple reference account of proper names has point, it is not clear that much credence can be put upon his criticism of the view that "a=b" says that what 'a' names is also named by 'b'. The contrast between Russell and Frege is brought to light, but any other contrast is unclear. But on reflection

it is not surprising that Frege's criticism of the "naming" view should fail, because Frege's own account of proper names in terms of sense and reference is very similar to the "naming" view.

We have seen how the sense of a name, for Frege, consists not merely in its reference, but in a particular way of identifying an object as the referent of the name. At the risk of suggesting something similar, it is clear that one common way of identifying the object to which a name refers is to find out which object has been so named. Given that there is a syntactic category of names, when an assertion is made using one or more of these names it is just assumed that each name used has been given to something. If a name is given to something or someone then the naming relation has been established between some one element of the category (or set) of names and a thing, place or person.

Name giving and acquaintance

Several things need to be noted about the naming relation. First, although ideally the naming relation is a function from a domain of names (the arguments) to a domain of what is named (the values), in natural languages this is not the case. There are, for example, many people who have the same name. Nevertheless there is a tendency in everyday life for people to be given names which will bring the ordinary language naming relation closer to the ideal. So we get the notion of a person's "full name", a notion which seems to give expression to the hope that no two people will have the same full name. Army serial numbers work this way also.

Secondly, the act of giving something a name is quite primitive. But I agree with Geach when he says,

John Austin seems to have held that naming is a momentous act, which just not anyone casually can perform; it would take the right person in the right circumstances using the right performative formula; and I am not at all sure that he would have counted me a validly ordained namer, or my baptismal formula for Pauline as a valid sacramental form. Well anyhow, I claim the rights to refer to any young lady of my acquaintance by the name "Pauline" for the course of this discussion; and I think the difference between such use of a name pro hac vice and the more official conferment of a name is only of legal or anthropological, not of logical, importance.¹

Thirdly, we must not confuse the giving of the name with its subsequent role as an argument for the naming relation. At the same time it is vital to remember that names can be given in other than the official settings of such things as christenings, or the launching of ships, or the claiming and naming of territories by explorers. When logicians claim that genuine proper names must name some existing object or particular it seems to me that they are forgetting that there is a vast range of situations in which names are given. If the giving of a name does establish the naming relation between a name and something named then it will not be untoward to consider some cases of the giving of names.

There are three sorts of cases which are of interest. There are those cases where, in the presence of the particular, a name is given it. The best example is that of a christening. Here there is no doubt about either who is named or that the person so named exists. There are, secondly, those cases where, in the absence of the particular but in the knowledge, or at least the belief, that it exists, a name is given it. Such a case is the case of the giving of the name "Vulcan" to a planet which some astronomers believed existed in an orbit inside Mercury's. This is an interesting case,

¹P. T. Geach, Logic Matters. Basil Blackwell, Oxford, 1972. p. 154.

because once the name was given both those who believed Vulcan existed and those who didn't used the word "Vulcan" as a name. There are a range of variations in these cases through those who know that the particular exists, and those who claim to know, and those who believe truly, and those who believe falsely, to those who guess or postulate. There is no need to go through every variation.

Thirdly, there are those cases where, in the knowledge that what is to be named does not exist, a name is given. This is the case typified by the writer of fiction giving names to the characters, animals, objects and places in his work.

Several things need to be noted about these various cases of the giving of names. First, there is the interesting feature of ordinary language that one can take some new word, invent it as it were, and by the act of name-giving that word becomes part of the syntactic category of names. Frege's 'Ateb' and 'Afla' are cases in point. There are, of course, already a large stock of words in ordinary language which can be put together in various combinations to make up a full proper name. But, by name-giving a word becomes a name. Secondly, although in one kind of case the name giving goes with an acquaintance with that to which the name is given, in many cases there is no acquaintance with that to which the name is given. A name is often given to something which has been identified descriptively, and often the description is not really a definite description.

For Russell, the sorts of name-giving where there is lack of any acquaintance with that to which the name is given are not really name-giving. Furthermore, anyone who uses a word as a name must be acquainted with the bearer of that name.

A name, in the narrow logical sense of a word whose meaning is a particular, can only be applied to a particular with which the speaker is acquainted, . . .

We are not acquainted with Socrates, and therefore cannot name him. When we use the word 'Socrates', we are really using a description.¹

This follows from Russell's account of proper names. If the sense of a proper name is its referent and a speaker knows the sense of a given name, i.e. understands the name, then the speaker must know the referent of the name. To know the referent of a name is, for Russell, to be acquainted with it.

Russell's claim that we must be acquainted with a particular before being able to use a word as its name makes understandable his further claim that "the only words one does use as names in the logical sense are words like 'this' or 'that'."² For, "One can use 'this' as a name to stand for a particular with which one is acquainted at the moment. We say 'This is white'. If you agree that 'This is white', meaning the 'this' that you can see, you are using 'this' as a proper name."³ Of this approach Hintikka remarks, "Here one feels, something has gone amiss. Not only is it strange to call 'this' and 'that' names, it seems positively perverse to allege that they are our only proper names properly so called."⁴

Russell's requirement that we must be acquainted with the bearer of a name in order to use a word as a name means that, for him, we cannot use the word 'Socrates' as a name. "We are not acquainted with Socrates, and therefore cannot name him. When we use the word 'Socrates', we are really using a description. Our thought may be

¹Op.cit. p. 201.

²Op.cit. p. 201.

³Ibid. p. 201.

⁴J. Hintikka, "Existential Presuppositions and Existential Commitments". The Journal of Philosophy, Vol. 56, 1959. p. 126.

rendered by some such phrase as, 'The Master of Plato,'. . . ."¹

Geach emphatically denies this point of Russell's when he writes, "It is indeed essential to the role of a name that the name can be used in the presence of the object named as an acknowledgment of its presence. But equally essential to the name's role is its use to talk about the named object in absentia."² But Geach does agree that acquaintance is essential at some point: "There must in the first instance be someone acquainted with the object named."³ Once there has been this initial acquaintance then the name may be used to talk about the named object in absentia, and in perpetuum. "Plato knew Socrates, and Aristotle knew Plato, and Theophrastus knew Aristotle, and so on in apostolic succession down to our own times; that is why we can legitimately use "Socrates" as a name the way we do."⁴

Although Geach denies the extreme thesis put forward by Russell, he supports the central notion that acquaintance is essential to a word's becoming a name, or being properly used as a name. So, for Geach, since no one has seen Vulcan, the word "Vulcan" is not really a proper name. It was not a proper name even when some astronomers believed Vulcan existed, and now that it is fairly certain that Vulcan does not exist it is also quite certain that "Vulcan" is not a proper name. The same can be said for all those words which were (and are) supposed to be the names of fictional, mythical or imaginary beings.

¹Russell, op.cit. p. 201.

²P. Geach, Logic Matters, Blackwell, Oxford 1972, p.154.

³Ibid. p. 155.

⁴Ibid. p. 155.

Empty names

If logicians like Russell and Geach are correct most of what has been described above as name-giving is not really name-giving. For Geach, the giving of a name to something which does not exist is really the giving of a quasi-name. He says:

Suppose we hear of a man who dreams of the same girl night after night, as happened in a story of H. G. Wells; for convenience of conversation, we may say he dreams of Petronella every night, without either committing ourselves to the view that there is a real live girl he is dreaming of, or meaning that the name "Petronella" is the name he gives the girl in his dreams. For us, "Petronella" is then functioning not as a name but as a quasi-name.¹

and also:

Names and quasi-names are of course grammatically proper nouns.²

It seems to me that Geach is trying to have his cake and eat it. He wants to say that real names designate only existing things, but since names like "Petronella" (he says: 'the name "Petronella"') are bestowed on characters in dreams he will have quasi-names. It is a distinction without a difference for which he can give, on his own word,³ no satisfactory account. But above all, he cannot stand imaginary entities. "I allow no such entities as imaginary girls in my universe of discourse."⁴ Geach is not alone in this prohibitionist attitude to imaginary entities.

Russell's attitude to the bearers of names is quite unequivocal, indeed ruthless, as we have already seen.⁵ For in the long

¹Op.cit. p. 161.

²Ibid. p. 163.

³Ibid. p.165.

⁴Ibid. p.156.

⁵Supra p. iv

run Russell's line on proper names leads to the virtual elimination of singular terms and the almost exclusive use of definite descriptions in his logic. But the view need not be pressed that far.

Frege is in general agreement with Russell. In a properly constructed formal language "it must be impossible to form a proper name lacking a reference."¹ Dummett explains that, for Frege, a sentence containing a proper name without a referent was a sentence without a truth-value. And "it is impossible to give any coherent account of the functioning of a language in which it is possible to construct well-formed sentences which lack truth-value."² Natural language is imperfect, and we make do with it. There are proper names in natural language without reference, but we understand what is said in spite of being unable to give truth-value to the sentences in which empty names occur.

Frege says, "A logically perfect language should satisfy the conditions; that every expression well constructed as a proper name out of signs already introduced shall in fact designate an object; and that no new sign shall be introduced as a proper name without being secured a reference."³ For example, Frege insists that in arithmetic one does not create numbers by definition and then give them names such as "zero", "one", etc. "Zero" is a proper name, but "Only when we have proved that there exists one object and one only with the required property are we in the position to give this object the proper name 'zero'."⁴ We must secure a reference for the name.

¹Dummett, *op.cit.* p. 167

²*Ibid.* p. 167

³*Op.cit.* p. 145

⁴*Ibid.* p. 145.

Even though Frege held to the desirability of every name's referring to an object, in practice, Frege's doctrine of sense and reference enabled him to cope with the situation where a name did not have a referent. Frege's solution for the problem of what it amounts to to use a name like "Vulcan" is to say that, as Dummett puts it,

Such an expression has a sense because we have a criterion, perhaps quite sharp, at any rate at least as sharp as for most names having a genuine reference, for an object's being recognised as the referent of the name: but it lacks a reference, because as a matter of fact there is nothing which would identify any object as the referent of the name; there is no object which satisfies the condition determined by the sense for being its referent.¹

Dummett concludes that Frege "had no need to postulate any realm of shadowy non-existent objects which yet had being, and could therefore be talked about and were talked about whenever we used one of these empty proper names."² In practice then Frege did not go the way of Russell and Geach and declare the names of Vulcan, Petronella, Pegasus and Pickwick not to be names at all. Frege accepts such names for what they are, and in as much as he does this his account is that much more intuitively acceptable.

But, once again, in Dummett as in Geach, Quine and others, we see this disparaging reference to "the shadowy realm of non-existent objects." But there is nothing shadowy about Pickwick, Pegasus or Cair Paravel. There are quite clear accounts of who, what and where they are. But of course, that is not what Dummett and the others are talking about, yet there does seem to be some lack of clarity. It is one thing to guard against the confusions which can

¹Dummett, *op.cit.*, p. 160.

²*Ibid.* p. 161.

arise when a name is used and it is not clear whether what is named exists or not, it is another thing to say that we cannot, in some logical sense, name non-existent objects. As Hintikka says of this view:

The criteria by means of which we recognise an expression as being a proper name do not involve ascertaining that there is a unique person (or object) to which it refers. Otherwise it would be a logical and not merely a factual mistake to be deceived by a pseudonym. The peculiarity of 'Bourbaki' will not lead us to repudiate its namehood. Empty names are still called names;. . .¹

Our ordinary language includes amongst the things of which we we speak a myriad of non-existent objects, but we are able in ordinary language to distinguish between real and imaginary, existent and non-existent. Why should we not then be able to do the same in logic?

This question is asked, not simply to throw down some sort of logician's gauntlet, but because there is a problem, one solution to which demands that we be able to do some such thing in our formal systems. The problem can be posed in the form of a question. In posing the question I will use the letters 'EP' for "existential presupposition", and take it that names interpreted in an EP way are names interpreted to be non-empty. The question is: If we are given a formal language which is to be interpreted in an EP way with respect to names, is it possible to consider situations in which those named individuals did not exist?

In the chapter on free logic I hope to show that the best way of considering situations in which named individuals do not exist is to abandon an EP interpretation of names, because the EP interpretation of names makes for considerable difficulty in the consideration of such situations.

¹"Existential Presuppositions and Existential Commitments", p. 127.

The sense of a name

We shall also explore the notion that words are entered into the syntactic category of proper names by their being given as proper names and that what Dummett calls "the criterion for an objects' being recognised as the referent of the name" is given in some basic sense at the name-giving -- sometimes by the circumstances, as with a baptism; sometimes by an author in the form of a description; sometimes by stipulation, or some other means. But in many cases there is no criterion which would satisfy a demand for a definite description other than saying that a is just the item to which the name 'a' was given. It does make sense simply to ask, when looking for the person who is called 'a', "Who is a?" We shall explore the implications of assuming that if we see the naming relation as the final irreducible criterion for an object's being recognised as the referent of a name, then we can interpret the notion of the sense of a given name as the relation established by that name's having been given to something.

It is possible to give a formal account of the naming relation and the sense of a proper name as outlined above. This is done in detail in Appendix I. Use is made there of the work of Church¹ and Rennie.² The formal system developed there relies on the basic idea that there are four categories of items, they are truth values, individuals, possible worlds, and syntactic items. The logic there is developed in terms of the relationships of a functional

¹A. Church "A formulation of the Simple Theory of Types", The Journal of Symbolic Logic Vol. 5, 1940, pp.56-68.

²M. K. Rennie, Some uses of type theory in the analyses of Language. Australian National University, Canberra, 1974.

type which can exist between these categories and categories built upon them. Our main interest is in the set of functions from names to individuals.

If a name is given to an individual, say 'a' is the name of a, then the relation between the name and the individual can be represented set-theoretically by $\{ \langle \underline{a}, 'a' \rangle \}$.¹ It can also be represented by $\{ \langle \text{the individual named by 'a'}, 'a' \rangle \}$. Either of these can be treated as the sense of 'a', and it is quite clear that the sense of 'a' is distinct from the sense of 'b' provided that 'b' is a different syntactic item to 'a'.

When a word is used as a name it is placed in a relation to that which it names. It is, of course, an item in the category of syntactic items, but within that category it becomes a member of the set of names. This is not to say that the sense of a name is "its having been assigned whatever reference it has been assigned",² as Wiggins says. It is the result of its having been assigned a referent, the setting up of the designation relation. Once the word has been assigned its referent it then has a sense. Indeed, Wiggins goes on to indicate just this when he says, "If standing for its referent is what a proper name's having a sense consists in, then there is no room for discrimination of sense arising from the particular circumstances under which a proper name may have been learnt. Such contingencies are overcome as they are overcome in the learning of the sense of any other sort of expression."³

¹The convention here is that the value of the function is to the left.

²D. Wiggins, "Essentialism, Continuity and Identity". *Synthese*, 23, 1974, p. 338.

³Ibid. p. 338.

But we are here emphasising the "standing for" rather than "that which" a name stands for. Wiggins emphasises the latter and says "If we attach the name \underline{n}_i to bearer \underline{b}_i and the name \underline{n}_j to bearer \underline{b}_j . . . and if $\underline{b}_i = \underline{b}_j$ then, whether we wish it or not, the sense of \underline{n}_i will be the same as the sense of \underline{n}_j ."¹ Yet, in a footnote, Wiggins says that "the thing designated is still in a different category from the sense of its name."² This is quite confusing. We have shown how we can preserve a partly Fregean notion of sense which is different from reference, and yet not dependent on the how each person comes to learn which object is the referent of a given name. We do not have to invent a new category of senses for names.

But, although we have provided an account of the sense of a proper name, this might not be sufficient for our purposes because of the nature of the questions which raised the problem in the first place. Frege's original problem was not why 'a' and 'b' are different names, but why "a=b" conveys quite different information to "a=a".

Even before we deal with that question we need to ask whether or not we need to understand anything more about 'a' and 'b' than that they are names, that they belong to that syntactic category. In a sense, the answer is that we do not have to understand anything more about 'a' and 'b' than that they belong to the category of names. Nevertheless, as we have defined it, a symbol's being a name just is a symbol's being related to that which it names.

Given that some members of the category of syntactic items are going to be used as names we identify a word's being used as a

¹Ibid. p. 338.

²Ibid. fn. 29, p. 357.

name by its logical role in sentences. If a word's logical role is not clear then there will be ambiguity. In spoken English, "Twas brown" can be quite ambiguous. If a word's logical role is clear, and it is clear that it is a name, then its sense is clear.

If, then, to understand a word's being a name is to understand its sense, what more must we understand concerning "a=b" and "a=a"? If there is anything then it will be about the relation of identity. It is worth noting at this point that there is not the same pressure on answering the question "What is identity?" that there is on answering the question "What is a proper name?" There are questions about whether identity is a relation or not, but these are in a different category. We are interested in the question of how a person's understanding of "a=b" differs from their understanding of "a=a" in such a way that the truth of one is determined in a way specifically different to the steps taken in the other.

That difference can be explained in terms of the sense of 'a' and the sense of 'b'. Whereas we understand "a=a" to be logically true because what is named by 'a' must be identical to what is named by 'a', it is clear that what is named by 'a' need not be identical to what is named by 'b'.

CHAPTER TWO

FORMALIZING AND SINGULAR TERMS

The elimination of singular terms

Before proceeding further with the account of proper names in terms of the naming function we must take account of the view that there is no need for any proper names in formalized or ideal languages. Quine claims that it would be better to eliminate names and to reparse simple singular terms as general terms, and so, there is no real need for any account of logically proper names.

Quine's arguments for this course of action fall under two main headings. On the one hand he argues that there are all kinds of problems which arise from the various uses of constant singular terms or proper names which would not arise in reparsed singular terms. On the other hand he argues that by reconstructing the category of names as subordinate to that of general terms we lose nothing. Though he does acknowledge that "there is a feeling that with reparsing the names as general terms we forfeit part of their meaning, viz. the purport of uniqueness."¹ But it is only a feeling.

We will take up Quine's arguments about the problems which arise from the various uses of singular terms when we are discussing free logics. At this point it is important to deal with his other set of arguments. For if proper names do not really need to be taken into account as such in formalized languages, but proper names can be accounted for by reconstructing them, then there is no need for any account of proper names apart from an account of general terms (and quantifiers, bound variables and identity in Quine's case).

¹W. V. O. Quine, Word and Object, M.I.T. Press, Massachusetts, 1960, p. 182.

Quine suggests that we can replace constant singular terms either by naming predicates or by definite descriptions. He proposes that in either case we retain only one kind of singular term, the bound variable. We turn first to his proposal for naming predicates.

Naming predicates

He proposes that when we want to formalize "Socrates is wise" we do not put it as "Ws", where "s" means "Socrates" and "W..." means "is wise." Instead, we should let "S..." mean "...is Socrates", and formalize to

$$(\exists x)(Sx \ \& \ Wx)$$

The proposal is that we take "Socrates is wise" to mean that of a set of individuals there is at least one which is both named "Socrates" and is wise. Clearly, we can point out that in this proposal, the "at least one" takes away the uniqueness presupposition.

So Quine suggests that the predicate "...is Socrates" is a uniqueness predicate. Just as "...is a cousin of..." is symmetrical and "...is a part of..." is transitive, so "...is Socrates" or "...is named 'Socrates'" is the property of one and only one individual. As Strawson puts it, "Instead of 'Socrates swims' we have 'Something is uniquely Socratic and swims' or 'Something which is uniquely Socratic swims'. Instead of 'Socrates' tout court we have 'Something which is uniquely Socratic'. As far as this language is concerned, names (and subject-terms in general) are to be seen simply as abbreviations for such constructions as these".¹

Now if a set of such distinct predicates were to be

¹P. F. Strawson, Subject and Predicate in Logic and Grammar, Methuen, London, 1974, p. 12.

introduced then we could claim that all Quine has done has been to re-introduce constant singular terms into the formal language. Even though they have been introduced in predicate form the same singularity of reference would be explicit in the primitive basis of the notation.

Quine himself says that "the distinction between singular and general terms is vaguely that a singular term names or purports to name just one object, though as complex or diffuse an object as you please, while a general term is true of each, severally, of any number of objects."¹ Also he says that "'Pegasus' counts as a singular term though true of nothing, and 'natural satellite of the earth' counts as a general term though true of just one object. As one vaguely says, 'Pegasus' is singular in that it purports to refer to just one object, and 'natural satellite of the earth' is general in that its singularity of reference is not something purported in the term."²

So, on his own account, since naming predicates would purport to refer to but one object they would have to be counted as singular terms. And just in case it is responded that naming predicates were to be in the same syntactic category as any other predicate, it can be pointed out that the kind of logical differences that would differentiate naming predicates from other predicates are precisely the differences which would differentiate constant singular terms from predicates.

We have already taken note of the referential purport of naming predicates. A second point can be brought out by a consideration of the following two formulae:

¹Word and Object, pp. 90-91.

²Ibid. pp. 95-96.

$$(a) \quad (\underline{x})(\underline{y})(\underline{x} = \underline{s} \ \& \ \underline{y} = \underline{s} \ . \supset \ . \ \underline{x} = \underline{y})$$

$$(b) \quad (\underline{x})(\underline{y})(\text{uniquely-}S\underline{x} \ \& \ \text{uniquely-}S\underline{y} \ . \supset \ . \ \underline{x} = \underline{y})$$

(a) is clearly a logical truth, and so must (b) be a logical truth if naming predicates are as Quine suggests. Now (b) is not a logical truth in general for predicates. In other words, the following is not a logical truth of quantification theory:

$$(c) \quad (\underline{x})(\underline{y})(F\underline{x} \ \& \ F\underline{y} \ . \supset \ . \ \underline{x} = \underline{y})$$

All of this indicates that one can eliminate constant singular terms in favour of uniquely referring predicates, but since the semantics for both are the same in principle, and the semantics for uniquely referring predicates will be different to the semantics for general term predicates, it looks as though Quine has simply replaced one syntactic category with another but left the semantics the same. There just does not seem to be any point in it. Also in terms of the sort of account of proper names that we have been developing there seems to be even less point. We have been developing the idea that the sense of a proper name is the relation which the name has to that which it names. Quine's moves with naming predicates accord with this account of the sense of proper names. So he seems to have changed nothing but the symbols. Quine himself, says, about the general term "is Socrates", that "such general terms might on that very account be denominated, more particularly, names".¹

Reference by description

Although Quine has suggested the device of a naming predicate to eliminate proper names, that device is not the only one suggested. The other important suggestion is to treat 'Socrates',

¹Quine, op.cit. p. 182.

for example, as a general term with no uniqueness characteristics, and then "'Socrates' as a singular term can be defined as ' $(\exists x)(x \text{ is Socrates})$ ' on the basis of 'Socrates' as a general term."¹ This move to definite descriptions is followed by a Russellian programme of eliminating definite descriptions by definition.

So Quine can say that "Singular terms have been reparsed where simple, and dissolved where of the form of descriptions."² And after showing how one can get rid of definite descriptions as such, Quine remarks in a footnote that "By retracing the relevant reasoning of § 37 in addition to the above, the reader can see that the manner of eliminating descriptions here is really the same as Russell's despite differences of approach."³

By following this programme Quine is left with predicates, indefinite singular terms and quantifiers, identity and the logical connectives, such as negation and conjunction, as the basic constituents of his formal language. Neither constant singular terms nor uniquely referring predicates are amongst these constituents. These constituents reflect the two basic notions on which he grounds his notation from a philosophical point of view. They are divided reference and the identity of objects. We turn first to Quine's notion of divided reference.

Divided reference

According to Quine one of the main differences between constant singular terms and general terms is that constant singular

¹Ibid. p. 189.

²Ibid. p. 184.

³Ibid. fn. p. 184.

terms purport to refer to but one object whereas general terms do not. Constant singular terms have singular reference, but general terms do not purport to refer to but one object and so have divided reference. Quine says that the expression "... is a natural satellite of the earth", although it has in fact but one referent, does not purport to refer to but one object. We could, without contradiction, have a whole set of objects each of which could be referred to by the expression "... is a natural satellite of the earth."

Although it does seem strange to say that a general term refers to any thing, especially expressions like "... is red" or "... is wise", we can see something of what Quine is getting at here, but it is problematic. Nevertheless, our point here is that even if we grant Quine the distinction between singular and divided reference we cannot agree that singular reference can be treated as less basic than divided reference.

For Quine, not only general terms, but some other terms also have divided reference. In particular, there are some singular terms which Quine claims have divided reference. This emerges in the course of Quine's moves to eliminate constant singular terms.

In his programme of eliminating names and definite descriptions Quine retains what he calls "indefinite singular terms". What this amounts to is that he retains bound variables¹, which do not stand alone, but are bound by a quantifier. The quantifier, either 'E' for 'At least one' or 'V' for 'Every', and variable together constitute an indefinite singular term.² For example, if we take 'F...' to mean '...is wise', then ' $(\exists x)Fx$ ' means 'Something is wise'. The ' $(\exists x) \dots x$ ' constitutes an indefinite singular term

¹Ibid. p. 178.

²Ibid. p. 162-163.

which is translated as 'Something ...'

Of these singular terms Quine says, "indefinite singular terms do not designate objects",¹ apparently meaning that they do not name any one object,² although there will be an object with the property predicated of the indefinite singular term if the property is truly predicated. Indefinite singular terms are said to be "built upon general terms,"³ and do not purport to refer to just one object. Indefinite singular terms, like general terms, have divided reference.

Divided reference, together with identity are basic for Quine because he holds that once a person has mastered these notions he has mastered the scheme of enduring and recurring physical objects. And things true in such a scheme of objects can best be expressed in a formal notation based upon divided reference and identity. So, if we can give good reasons for holding that one cannot master divided reference and identity without grasping uniqueness of reference, then we could argue that the elimination of singular terms severely impoverishes a language which is supposed to be adequate for the expression of truths about the objects in the scheme proposed.

So first let us consider what Quine has to say about this scheme of enduring and recurring physical objects. The scheme is derivative in a strong sense. What are primary are the stimulations which a person receives. These stimulations resemble each other and differ from each other. A person learns to respond in a general term fashion to stimulation. This is so because of a recognition of resemblances. A child "must, so to speak, sense more resemblance

¹Ibid. p. 146.

²Ibid. p. 113

³Ibid. p. 163.

between some stimulations than between others. Otherwise a dozen reinforcements of his response 'Red', on occasions when red things were presented, would no more encourage the same response to a thirteenth red thing than to a blue one; and a dozen reinforcements of his response 'Mama', on occasions dominated by the mother's face at various angles, would be just as inconsequential."¹

In the course of time, building on the recognition of the substantial sameness of stimulations a child has to "get on" to divided reference, and hence to have mastered the scheme of physical objects. Quine says that, "To what extent the child may be said to have grasped identity of physical objects (and not just similarity of stimulation) ahead of divided reference, one can scarcely say without becoming clearer on criteria."²

So, for Quine, there is a distinction between the identity of objects and the divided reference of general terms. With these two notions a person may name an object, and hence have mastery of a singular term.

Quine's use of the 'Mama' example shows how he sees a child's development of the conceptual scheme. At some point there is a shift from a certain response to similar stimulations to a certain response to the same object. This shift reflects a grasp of both what it is for objects to be the same and what it is for a term to divide its reference. So, instead of 'Mama' being a response to similar stimulations, and thus, in a sense, a general term, 'Mama' becomes the name of an object. For the child the one word refers always to the same object, and hence is a name. Other words which continue to divide their reference become general terms.

¹Ibid. p. 83.

²Ibid. p. 95.

So, in the context of several utterances such as "Mama sings", "Papa sings", "Mama cooks", "Papa does not cook", ... the child is showing that one and the same object is similar to another object in certain ways, and dissimilar in certain ways, as well as showing that he now can use the physical object scheme.

Both for mastery of the physical object scheme and for the ability to be able to name objects, Quine says that a person must have grasped the notion of the identity of physical objects and the notion of a general term's dividing its reference.

One way of looking at the person's grasping of or getting on to the notion of the identity of physical objects is to see the person tacitly assuming that the world is made up of distinct objects, or that there is a series of distinct objects, or that the world about the person can be seen as a collection of distinct objects. This tacit assumption gives sense to divided reference in both the bound variable and general term senses. To say this is not to say that quantifiers and general terms derive all their meaning from the notion of the identity of physical objects, it is just to insist that this notion is essential.

All of what has been said so far has been focussed on divided reference. We now turn to identity and its relationship to divided reference. In the course of the discussion we shall take up criticism of Quine's point of view.

Identity and divided reference

Even within Quine's own conception of how one comes to grasp the conceptual scheme of physical objects we can point out the basic inadequacies of his notion of identity and its relation to divided reference. In fact he says very little about identity, but

he does speak about how one comes to grasp the notion of 'same x '. He suggests that this notion is grasped in a fashion analogous to the way in which one grasps divided reference.

By a process of binary ostension one learns 'same-person'. This term goes with simultaneous or closely consecutive presentations in pairs. It proves to apply whenever both presentations are appropriate, "for example, to 'Mama' and whenever both are appropriate to 'Daddy'".¹

Although, as Quine says, his example is unrealistic and merely cited to demonstrate some sort of analogy, we must take it seriously in some sense otherwise it is mere phantasy. But his example is not only unrealistic, it seems to have the logical order the wrong way around. Quine's thesis is that the temporal order, or at barest minimum the logical order, is that first a person grasps the notion of identity and of a term's dividing its reference. Then (second) one grasps the notion of 'same x ', perhaps at the same time as one grasps the notion of identity and of a term's dividing its reference, and then (third) one can name objects.

But in the example given, one learns "same-person" because it applies to closely consecutive presentations of 'Mama' or 'Daddy'. Although it is not indelibly clear, it looks very much as though the third and second steps, at least, are round the wrong way. It even looks as though a person is, in Quine's example, learning identity and divided reference on the basis of having learnt to name Mama or Daddy, or to recognise these persons as so named.

Also, Quine himself, in his section on identity, says that "statements of identity that are true and not idle consist of

¹Ibid. p. 94.

unlike singular terms that refer to the same thing."¹ Now it is not much help for Quine to say that we base assertions of identity on the notion of same physical object, because he has already said that we only master the notion of same physical object when we master the notion of identity. Perhaps one masters them together as one "scrambles up an intellectual chimney, supporting himself against each side by pressure against the others."²

If we accept the chimney image the least that can be said is that identity and naming go together, that is, singular reference is essential for identity. And so, what is needed in order to grasp the scheme of recurring and enduring physical objects is mastery of both singular and divided reference.

Nevertheless, it can be responded that once the scheme of recurring and enduring physical objects has been mastered only identity need be included in a language which is to be used to state truths about these objects. An understanding of singular references' being essential for an understanding of identity simply means that the use of identity implies an understanding of singular reference. Although this lack of singular terms impoverishes the language, we can do without them. As Quine says, "having got up to here, we kick away the ladder of definition and singular description by which we climbed."³

¹Ibid. p. 117.

²Ibid. p. 93

³W. V. O. Quine, "To Grice" in Words and Objections: Essays on the work of W. V. Quine, Eds. Donald Davidson and Jaakko Hintikka. Reidel Dordrecht, 1969, p. 326.

Singular Reference

But just what does this claim of Quine's amount to in terms of a formal language and its semantics? It is certainly true that in first order predicate calculus one can do without a set of individual constants. Church's system, F_p^{II} , is a classic case of just this. But, as has already been pointed out, the lack of individual constants is made up for in the semantics. A careful distinction is drawn between bound and free occurrences of individual variables, and then the value of a variable occurring free in a formula is given as a single item in the domain of quantification. Singular reference is preserved in the semantics by this distinction between free and bound occurrences of individual variables.

What this claim must amount to is that there shall be no free occurrences of individual variables in the formal notation most to be desired. This then raises the question of what will be the semantics for quantified formulae. Are the semantics to involve singular reference? For standard set-theoretic semantics the answer on prima facie grounds would be 'yes'. If we give the semantics for $(\exists x)Fx$ in the usual way then we have

$$(i) \quad \text{Val}(F) \in \mathcal{P}(D_L)$$

$$(ii) \quad \text{Val}(\exists x)Fx = 1 \equiv \text{Val}(F) \neq \emptyset.$$

But how do we decide whether or not $\text{Val}(F) \neq \emptyset$? We decide it by seeing whether or not there is some one member of $\text{Val}(F)$ to which we may make singular reference. If it is impossible to make singular reference, $\text{Val}(\exists x)Fx = 0$.

This also turns out to be the sort of argument which develops if we resort to our formal type theoretic metatheory.¹ It turns

¹See Appendix I.

out there also that we are driven back to the semantics for quantification.

Geach¹ has developed a parallel argument with much the same conclusion. It is parallel in the sense that he does not develop it in the context of the semantics for quantifiers, but rather, in the context of the sort of natural deduction decision procedure followed by Quine in Methods of Logic² and elsewhere. In discussing the rule of Existential Instantiation (EI) Geach says:

The premise " $(\exists x)Fx$ " gives us what is supposed to be true in some instance; we make believe that we can actually cite an instance w , and then, if a conclusion follows from this make-believe citation of an instance, " Fw ", regardless of which instance w may be, that conclusion is deemed to follow from " $(\exists x)Fx$ ". Thus, since quantification theory can be formulated in this natural-deduction style, it necessarily gives us 'possible' proper names, namely the dummy letters like " w " used in natural deduction as if they were proper names.³

This resort to 'possible' proper names is the resort to singular reference. Geach goes on to say, "This deprives of all philosophical significance Quine's neo-Russellian treatment of proper names as disguised descriptions."⁴

For both standard semantics and in natural-deduction methods there is the explicit assumption of the possibility of singular reference. It is almost impossible to see how one can avoid resorting to singular reference, or some substitutional surrogate for it, such as substituting proper names. If Quine kicks away his ladder of singular reference then he will lose touch with the Quinean ground of being -- the range of bound variables.

¹Op.cit. pp. 139-146.

²W. V. O. Quine, Methods of Logic, Holt, New York, 1950.

³Op.cit. p. 143.

⁴Ibid. p. 143.

CHAPTER THREE

NAMING ACROSS POSSIBLE WORLDS

We now turn to modal predicate logics in order to see whether the proposed theory of proper names is of any use in clarifying a range of problems such as the problem of transmundane identification and the problems of the substitutivity of identicals. These problems will be approached, not by the immediate consideration of a range of modal predicate logics, but by the consideration of some conjectures and then by the construction from the ground up, as it were, of a range of logics. We shall delay the introduction of modal operators somewhat. After the introduction of modal operators we shall consider the uses of definite descriptions in modal predicate logic, especially considering the relationship between definite descriptions and proper names.

Time and History

Writers of science-fiction have often used the idea of someone's going from this universe to some other universe which, although it is similar to this one, is different in some singularly important way. In many cases the source of the difference is to be found in some historic event which has turned out differently to what has actually happened. Perhaps Napoleon wins Waterloo, or Hitler is assassinated in 1938, or Churchill dies in 1940, or Hannibal conquers Rome, or Socrates wins his court-case.

The idea that history could have been the same up to some particular point and then have diverged, has obvious intuitive relations to either Kripke's notion of sets of possible worlds or

Hintikka's notion of model systems. Indeed, Hintikka is quite explicit about this very thing when he says:

What I take to be the distinctive feature of all use of propositional attitudes is the fact that in using them we are considering more than one possibility concerning the world.¹

We can set up a model system to reflect the difference between the actual and some possible (fictional) world. The actual world, g , can be (partially) described by some set of propositions, μ_g , such as would be found in a history text. A second set of propositions, μ_f , would describe a fictional world, f . There could be much in common between the two sets, especially if the fictional account asserted that everything before some particular event was the same as in the actual world. The particular event, let us say, is Hannibal's crossing of the Alps, and in the fictional world Rome falls to Hannibal and Carthage dominates the Mediterranean states for centuries. Given such a story of a Carthaginian Mediterranean, then there would still be many propositions in μ_f which would be in μ_g , but many which would either not be in μ_g or whose negation would be in μ_g .

In the context of our Punic story we can ask about the relationship between the individuals in g and f , treating this as an instance of the general problem of the relationship between individuals from world to world. We can ask, do μ_g and μ_f show us possibilities, as far as what could have happened when Hannibal was in Italy, one of which was fulfilled and the other not; or do μ_g and μ_f show us two quite distinct possible worlds which are remarkably similar in some respects but quite different in others? This question is relevant to conjectures about what might have been the case, both in the

¹J. Hintikka, Semantics for propositional attitudes, in Philosophical Logic, Eds. J.W. Davis et al. Reidel Dordrecht, 1969, p. 24.

situation when we don't know and are looking for the likeliest possibility, and also when we are discussing possibilities contrary to fact. From such conjectures questions also arise concerning the identification of individuals across possible worlds. The fictional and the philosophical are closely related here.

Pursuing our Punic story, consider the following: Let us suppose that Wells¹ invents a time-machine and "goes back" to Hannibal's invasion of Italy. Then by means of his modern skills, he helps Hannibal to overcome Rome and defeat Scipio. As a consequence of this Carthage dominates the Mediterranean, and modern history is radically different, and Wells does not invent his time machine. In other words, we first assume that if he invents his machine then he helps Hannibal, then we discover that if he helps Hannibal he does not invent his machine. We can conclude that he did not invent his machine and go back and alter history so that his own machine could not be invented.

Let us consider two of the ways of escaping the conclusion of our time travelling conjectures. These ways of escaping will have to involve an alteration in the story, or its framework, or some presupposition about "going back" in time.

The first way of escaping is to postulate alternative possible worlds which are quite disjoint. Since the world in which Wells makes his machine does not have Wells helping Hannibal to win, then Wells cannot arrive, or appear, at some point in its past because he does not. So we assume that Wells' machine actually takes him to some other possible world, an alternative one to this one,

¹In H. G. Wells' story "The Time Machine" (1895) the time-traveller is not Wells, nor is the story told by the time-traveller. I have just borrowed the surname. Let us name him "O. H. Wells".

one in which a Mr. Wells appears and helps a general called Hannibal. The fact that the history of that world is the same as the history of this one up to the point when Wells appears is but an amazing coincidence.

This escape route is used by various writers. The important feature of this kind of story is that the Hannibal of this world and the Hannibal of that world are not one and the same person. Neither is the Socrates of this world nor the Socrates of that world one and the same, nor anyone else, except the time-traveller. Even if the descriptions within μ_g and μ_f are the same in every way, the people, places, things etc., are distinct with only the one exception, Wells.

So the natural language name "Socrates", though it occurs in sentences expressing the propositions in both μ_g and μ_f , suffers from the same kind of ambiguity as any natural language name which names two persons. If we formalize the expression of the propositions in μ_g and μ_f within one formal system, then we will have to use distinct individual constants for the two Socrates. We can do as we would in the case of two Mr. Jones in one world, that is, use any two constants, or we could do something a little more intuitively acceptable, that is, use the same letter and subscript but with a superscript to indicate the world. "Socrates" in μ_g might be represented by ' s_1^g ', and "Socrates" in μ_f by ' s_1^f '.

Hughes and Cresswell make a recommendation somewhat similar to this latter alternative when setting out symbols for a quantified modal logic with contingent identity. Similarly, we could take a denumerable list of individual constants: $a_1, b_1, c_1, a_2, b_2, c_2, a_3, \dots$, and give to each the superscript \underline{n} when it named an individual in the world described by μ_n .

The second way of escaping relies upon the notion of "branching" worlds. This idea is that there is but one world up until the moment before Wells appeared. Upon that instant there begins a branching into two possible worlds, the one which we call "the real world", the other "the time traveller's world." μ_g would be the description of the world up to and then along the 'real' branch, μ_f the description up to and then along the 'time traveller's' branch.

The important feature of this kind of story is that, unlike the first where only Wells is common to both worlds, there are many individuals common to both.

Individuals, worlds and identity

At this point we can generalize about the individual constants and the individuals they designate relative to each μ_n . If there are denumerably many possible worlds, each (partially) described by a set of propositions, μ_n ($n \geq 1$), then the names used in expressing the propositions could designate individuals such that either:

- I.) There would be but one set of individuals each of whose members would be common to each possible world, or
- II.) There would be one distinct set of individuals each of whose members would be common to each possible world, and one distinct set of individuals for each and only one possible world, or
- III.) There would be sets of individuals whose members would be common to the members of various sets of possible worlds, ranging from unit sets of possible

worlds to the whole set, or

- IV.) There would be as many distinct and exclusive sets of individuals as possible worlds with but one set for each possible world.

In the first three cases we have the situation where individuals are common to several worlds. This is what was suggested above in both ways of escaping the time travel paradox. For one there was only one individual common to both worlds, in the other there were many. Case IV is just the situation suggested by Lewis's Counterpart theory.¹

These cases raise a problem about what can be expressed about common membership of various worlds in any formal language we might construct. If we set up a formal system consisting of a model system, Ω , of model sets, $\mu_n (n \geq 1)$, so that $\Omega = \{u_1, u_2, u_3, \dots\}$, then although each μ_n can be taken as a (partial) description of a possible world we must decide where the "trans-world" identities are to go, or even whether we are to tolerate the expression of such propositions in the formal language. If we were to decide not to tolerate such trans-world identities, for whatever reasons, and we were to superscript individual constants in the manner suggested above, then it is fairly clear what would happen in our formal system. We would simply write the formation rules so that only individual constants with the same superscript would occur in the one formula. Then, those formulae with the individual constants superscripted with n could go only into μ_n .

If we decide to tolerate the expression of transworld identities then suitable provision will have to be made in the formation rules, and suitable conditions set out for the membership of members of Ω . It would be possible either to create a distinct

¹D. Lewis, "Counterpart Theory and Quantified Modal Logic", The Journal of Philosophy, 65, 1968.

transworld identity set as a member of Ω and to restrict transworld identities to that distinct set of formulae, or to allow transworld identities to be members of some or any of the μ_n in Ω . Because of certain difficulties which would arise with the former possibility in the long run we here adopt the latter policy.

It must be noted that, in general, the individual named by ' a_i^n ' need be in no way similar to the individual named by ' a_i^m '. The use of the same letter and subscript with differing superscripts is no more than a typographical concession to the use of names in natural language. The individual constants are the symbols as a whole and ' a_2^1 ' is as different to ' a_2^6 ' as would ' a^1 ' be to ' a^6 '.

Also, in what follows, we shall assume that Ω is a set of sets of formulae of some formal system. If the members of were maximal consistent sets of formulae then each member could be called a complete novel¹, and a possible world is simply what is described by the propositions expressed by the members of a complete novel, except for those members which are trans-world identities.

Quantification, transworld identity, and no modal operators

We will now proceed to the construction of four formal systems, without modal operators, to reflect each of the cases I to IV. It is important to be clear about the presuppositions in terms of which these systems will be built. Our first assumption is that an individual constant designates one and only one individual. It does not matter in which formulae an individual constant occurs, or in which μ_n in an Ω the formulae occur, a constant designates only

¹J. Hintikka, "Individuals, Possible Worlds and Epistemic Logic", Nous Vol. 1 No. 1, p. 41.

one individual. Our assumption that whatever a constant designates it designates always means that we are assuming that constants are what Kripke calls "rigid designators."¹

The second of our assumptions is that quantifications, such as (E_x) and (U_x) , will have as their domain of quantification the set of individuals in the world (partially) described by the μ_n in which the quantification occurs. So, if μ_n describes world n , and $(E_x)Fx$ is a member of μ_n , then our reading of $(E_x)Fx$ as "something is F" presupposes that the something is of world n . To be quite explicit we should read

$$(E_x)Fx \in \mu_n$$

as "Something in world n is F." Similarly we should take

$$(U_x)Fx \in \mu_n$$

as indicating that everything in world n is F.

With these two presuppositions in mind we will construct the four systems $\mu^i Q = (1 \leq i \leq 4)$. These four systems are to reflect the cases I to IV respectively. We will begin by constructing $\mu^4 Q =$ to reflect case IV. We begin with $\mu^4 Q =$ because it is in many ways the simplest case. Once we have constructed all four systems, $\mu^i Q = (1 \leq i \leq 4)$, we will then introduce modal operators and construct a set of modal predicate logics.

Our procedure is unlike the usual procedure of taking some formal system and attempting to give it an interpretation. We are presupposing certain things about individual constants and quantifiers and constructing systems in accordance with those presuppositions. The systems will differ only with respect to one thing, and that one

¹S. A. Kripke, "Naming and Necessity" in Semantics of Natural Language, Eds. D. Davidson & G. Harmon. Reidel Dordrecht, 1972, pp. 269 ff.

thing concerns the relationships between domains of quantification as set out in cases I to IV.

As we proceed certain questions will arise, and will be dealt with. By the time we get to the point of introducing modal operators we shall have determined certain important questions of interpretation, and this should assist us with the modal systems. But above all, it will be seen that our presuppositions about individual constants will lead to reasonable and readily interpretable systems of modal predicate logic. Furthermore, in these systems certain problems concerning identity and description can be dealt with quite effectively.

So we now proceed to system μ^4Q which is to reflect case IV, where the sets of individuals in the worlds are quite distinct one from the other.

Formalizing

$\mu^4Q =$

Primitive symbols:

improper symbols: $\sim \supset U()$

propositional variables: $p_1, q_1, r_1, p_2, q_2, r_2, p_3, \dots$

individual variables: $x_1, y_1, z_1, x_2, y_2, z_2, x_3, \dots$

individual constants: $a_1^n, b_1^n, c_1^n, a_2^n, b_2^n, c_2^n, a_3^n, \dots (n \geq 1)$

m-ary predicate variables: $F_1^m, G_1^m, H_1^m, F_2^m, G_2^m, H_2^m, F_3^m, \dots (m \geq 1)$

predicate constant: =

Formation Rules:

1. A propositional variable standing alone is a wff.
2. If F^m is an m-ary predicate variable, and if a_1^n, \dots, a_m^n are m individual constants (necessarily distinct) then

$F^m a_1^n a_2^n \dots a_m^n$ is a wff.

3. If \underline{a} and \underline{b} are any individual constants (not necessarily distinct) then $\underline{a} = \underline{b}$ is a wff.
4. If \underline{A} is a wff so is $\sim \underline{A}$.
5. If \underline{A} and \underline{B} are wffs and all the individual constants occurring in \underline{A} and \underline{B} outside of atomic parts which are identities have the same superscript, then so is $(\underline{A} \supset \underline{B})$.¹
6. If \underline{A} is a wff and \underline{x} is any individual variable, then provided that all the individual constants in \underline{A} have the same superscript, then $(U\underline{x})(\underline{A} (\underline{x} // \underline{a}))$ is a wff, where $(\underline{A} (\underline{x} // \underline{a}))$ is the result of substituting \underline{x} for \underline{a} at zero or more occurrences of \underline{a} in \underline{A} .

T. Terminal clause.

Formulae constructed according to rules 1 and 2 are called "atomic wffs", and formulae constructed to rule 3. are called "identities". The "atomic parts" of any formula are the smallest well formed parts. The normal definitions of $\&$, \vee , $=$ and \exists are used. Also $(\underline{A} (\underline{X} / \underline{Y}))$ is the result of substituting \underline{X} for \underline{Y} at every occurrence of \underline{Y} in \underline{A} .

Model System:

A Model system Ω is such that

$$\Omega = \{ \mu_1, \mu_2, \dots, \mu_n, \dots \} \quad (n \geq 1)$$

and the members of Ω are model sets, i.e. sets of formulae. The conditions which determine the membership of any μ_n in a $\mu^4 Q =$ model system are

(C. μ_n) If \underline{A} contains any individual constant \underline{a}^i occurring outside of an identity and $i \neq n$, then $\underline{A} \notin \mu_n$.

(C.DD) If \underline{a}_j^i and \underline{b}_m^k are individual constants, $(i \geq 1)(k \geq 1)$, in any

¹

In other words: all the constants in \underline{A} and \underline{B} have the same superscripts other than those occurring in identities where different superscripts are allowed.

member of a member of Ω and $i \neq k$ then for every μ_m in Ω

$$a_j^i \neq b_m^k \in \mu_n$$

(Comment: these first two rules ensure that transworld non-identities occur in model sets. The second rule is called '(C.DD)' because it is the disjoint domain condition.)

(C.~) Not both $A \in \mu_n$ and $\sim A \in \mu_n$.

(C.~~) If $\sim \sim A \in \mu_n$ then $A \in \mu_n$.

(C.⊃) If $(A \supset B) \in \mu_n$ then either $\sim A \in \mu_n$ or $B \in \mu_n$ or both

(C.~⊃) If $\sim(A \supset B) \in \mu_n$ then $A \in \mu_n$ and $\sim B \in \mu_n$.

(C.E_n) If $(\exists x)A \in \mu_n$ then $(A(a^n/x)) \in \mu_n$ for at least one individual constant a^n .

(C.U_n) If $(\forall x)A \in \mu_n$ then if b^n occurs in any formula in μ_n then $(A(b^n/x)) \in \mu_n$.

(C.self ≠) $b \neq b \notin \mu_n$

(C.ⁿ) If $A \in \mu_n$ and $(a_i^n = b_j^n) \in \mu_n$ then $(A(a_i^n / b_j^n)) \in \mu_n$

(Comment: the rules, (C.~) to (C.ⁿ), are the standard rules for predicate logic with identity where the individuals in the domain of quantification for μ_n are designated by individual constants with the superscript n .)

(C. $\mu =$) If $A \in \mu_n$ and all the atomic parts of A are in the form of identities, then for each individual constant a_j^i , which occurs in A with $i \neq n$, $A \in \mu_i$, and if a_j^i occurs in any formula in μ_m then $A \in \mu_m$.

(C. $= =$) If $A \in \mu_n$, and all the atomic parts of A are in the form of identities and $a = b \in \mu_n$, then $(A(a // b)) \in \mu_n$.

(Comment: It is possible to condense this set of rules by, for example, removing the superscript restriction in (C.ⁿ) and then dropping (C. $= =$) altogether. But if we do this it will make it more difficult to compare the systems $\mu^i Q = (1 \leq i \leq 4)$. These last two rules ensure that any transworld non-identity in one model set

in a model system will occur in all the model sets in the model system and that any identities, or formulae constructed out of identities, will be consistent one with the other.)

All these rules are members of the set C^4 . Any model system, Ω , whose membership is determined by the members of C^4 is a $\mu^4 Q = (\Omega)$. In general we let ' $S(\Omega)$ ' stand for " Ω is a model system whose membership is determined by a set of rules for system S ", or for " Ω is an S model system."

A set of formulae, λ , is said to be satisfiable (consistent) in a system S if the set is included in some μ_n in some S model system. We let ' $S \text{ Sat } (\lambda)$ ' stand for " λ is satisfiable in S ." So we have

$$S \text{ Sat } (\lambda) \equiv (\exists \Omega)(\exists \mu_n)(S(\Omega) \& \mu_n \in \Omega \& \lambda \subseteq \mu_n)$$

If λ is satisfiable in S then we say that each member of λ is satisfiable in S , or just 'satisfiable' if the context makes clear which S we intend.

A formula is said to be logically true in S or self-sustaining in S if the unit set of its negation is not satisfiable in S and the formula is satisfiable in S . When the context makes it clear which system is under consideration we will simply say that a formula is self-sustaining. We let ' $S \text{ Self-sus } (\underline{A})$ ' stand for ' \underline{A} is self-sustaining in S .' So we have:

$$S \text{ Self-sus } (\underline{A}) \equiv S \text{ Sat } (\{\underline{A}\}) \& \sim(\exists \Omega)(\exists \mu_n)(S(\Omega) \& \mu_n \in \Omega \& \{\sim \underline{A}\} \subseteq \mu_n)$$

A formula will therefore be not self-sustaining in S if either the formula is not satisfiable in S or its negation is satisfiable in S .

In $\mu^4 Q =$ the following set of formulae is satisfiable:

$\{ W_{a_1}^1, (E_x)(F_x \& G_{b_1}^1), a_1^1 = a_1^2, a_1^1 = b_1^1, \sim p, (p \supset q), a_1^2 = a_1^2 \}$

This set could be included in a μ_1 in some Ω . The set could be read off as saying

- i. Socrates¹ is wise
- ii. Something is fast and Bill¹ is a philosopher
- iii. Socrates¹ is not Socrates²
- iv. Socrates¹ is Bill¹
- v. $\sim p, (p \supset q)$
- vi. Socrates² is Socrates.²

iii gives expression to the assumption that the domains for μ_1 and μ_2 are distinct (by (CDD)). But we do allow propositions such as vi.

The following set of formulae is clearly not satisfiable

$\{ W_{a_1}^1, W_{a_1}^2, a_1^1 = a_1^2, b_1^3 = b_1^5 \}$

The first and second formulae cannot be together in any μ_n (by rule (C. μ_n)); and both identities are ruled out by (C.DD) with (C. \sim).

This system and the others we construct are set out together in full and contrasted in Appendix II.

We show that a set of formulae, λ , is $\mu^4 Q =$ satisfiable by constructing on λ a model set μ_n in accordance with the rules in C^4 . If λ is not satisfiable it will prove impossible to construct on it a μ_n which accords with the rules in C^4 . To show that a formula is self-sustaining we show that the unit set of its negation is not satisfiable, but we must also show that the formula itself can be a member of a μ_n . The reason for this latter stipulation is that rule (C. μ_n) will exclude some formulae together with their negation.

The system, $\mu^4 Q =$, which has now been set out is supposed to reflect case IV as set out above. Here we assume that each possible world has a quite distinct set of individuals in it,

disjoint from the individuals in all the other worlds. But in order to give expression in the system to this assumption we have to abandon the idea that all the individual constants in a μ_n designate individuals only in the world described by μ_n . Nevertheless the rules for quantifiers indicate that the domain of quantification for any quantifier in a formula in any μ_n is the set of those individuals designated by individual constants with superscript n , and only those individuals. So we can say that each μ_n in $\mu^4Q =$ is related to its own domain of quantification, and that domain of quantification is the set of individuals in the world described by all the formulae in a μ_n , except for the trans-world identities and their negations.

In order to ensure that there are disjoint domains we have had to allow trans-world identities and their negations into the set of well-formed formulae. The disjoint domain rule (C.DD) then completes the task. If we allowed no trans-world identities (or their negations) into the set of well formed formulae and enforced the same sort of formation rule for identity as we did for predicates (rule 2.) then we could interpret all sets of formulae with just one domain.

We are assuming that each individual constant names one and only one individual, although each individual can be designated by more than one individual constant with the same superscript. If each individual constant names one and only one individual in just one set of individuals then we need (C. $\mu =$) in order to prevent the following model system from being consistent.

$$\begin{aligned}\Omega &= \{ \mu_1, \mu_2 \} \\ \{ p, q, a_1^2 = b_1^2, a_1^2 = c_1^2 \} &\subseteq \mu_1 \\ \{ p, q, a_1^2 \neq b_1^2, a_1^2 \neq c_1^2 \} &\subseteq \mu_2\end{aligned}$$

In μ_1 we have described a situation in which ' a_1^2 ', ' b_1^2 ', and ' c_1^2 ', all name the one individual, but in μ_2 we have described the situation

where this is not the case. Rule (C. $\mu =$) prevents such a situation arising by making sure that all four identities are consistent with each other.

$$\underline{\mu^3 Q} =$$

The system $\mu^3 Q =$, to reflect case III above, is different to $\mu^4 Q =$ in the following way:

The conditions for membership of a $\mu^3 Q = (\Omega)$ will omit rules (C. μ_n) and (C.DD) and add

(C.=) If $\underline{A} \in \mu_n$ and $\underline{a} = \underline{b} \in \mu_n$ then $(\underline{A} (\underline{a} // \underline{b})) \in \mu_n$

The amended set of rules is C^3 .

By omitting (C. μ_n) and (C.DD) from C^3 we allow for the individuals in the world described by μ_n being in a world described by another set μ_m . The quantifiers in both $\mu^3 Q =$ and $\mu^4 Q =$ have as their domains the sets of individuals that occur in the relevant worlds.

For example, if $(\underline{Ex})\underline{Fx} \in \mu_1$ and $(\underline{Ex})\underline{Fx} \in \mu_2$, the intersection of the domains of these quantifiers can be either empty or non-empty for $\mu^3 Q =$, but they must be empty for $\mu^4 Q =$. In $\mu^3 Q =$, the way to show that the intersection of the sets of individuals in two worlds is not empty is to have in the model sets which describe those worlds trans-world identities which indicate the individual or individuals common to both worlds. For example, if

$$\underline{a}_1^1 = \underline{b}_1^2 \in \mu_1$$

then we know that there is at least one individual common to both world 1 and world 2. Rule (C. $\mu =$) will ensure that

$$\underline{a}_1^1 = \underline{b}_1^2 \in \mu_2.$$

(C. $\mu =$) is to ensure that the overlapping of sets of individuals is expressed in all relevant model sets. If, as well as the above identity, we have $(\underline{Ex})\underline{Fx}$ in μ_1 and $(\underline{Ex})\underline{Fx}$ in μ_2 then we know that

the domains of quantification overlap.

$\mu^4 Q =$ is a sub-system of $\mu^3 Q =$. The formulae are the same for both systems. It can also be shown that the formulae satisfiable in $\mu^4 Q =$ are satisfiable in $\mu^3 Q =$.

Furthermore, we can construct particular model systems in $\mu^3 Q =$ which reflect case IV, and all the other cases also. To reflect case IV we first construct a set ν of non-identities as follows:

$$\nu = \{x^n \neq y^m \mid n \neq m\}$$

ν is the set of all transworld identities negated. We then construct a $\mu^3 Q = (\Omega)$ but we include ν in μ_1 in Ω .

One way to reflect case I is first to construct a set as follows:

$$\nu = \left\{ \begin{array}{l} a_1^1 = a_1^2, a_1^2 = a_1^3, a_1^3 = a_1^4, \dots \\ b_1^1 = b_1^2, b_1^2 = b_1^3, b_1^3 = b_1^4, \dots \\ c_1^1 = c_1^2, c_1^2 = c_1^3, c_1^3 = c_1^4, \dots \\ a_2^1 = a_2^2, a_2^2 = a_2^3, a_2^3 = a_2^4, \dots \\ b_2^1 = b_2^2, \dots \\ c_2^1 = c_2^2, \dots \\ a_3^1 = a_3^2, \dots \\ \vdots \\ a_n^1 = a_n^2, \dots, a_n^i = a_n^{i+1}, \dots \\ \vdots \end{array} \right\}$$

The formulae in ν indicate that the individuals in world 1 are in every world, and that if any individual is in any world it will be in world 1. For example, what is called ' a_1^1 ' in world 1 will be called ' a_1^2 ' in 2, and ' a_1^3 ' in 3, and so on. All individual constants will

appear in ν . We then construct a $\mu^3Q = (\Omega)$ but we include ν in μ_1 in Ω .

One way to reflect case II is first to construct a set as follows:

$$\nu = \{ \underline{a}_1^1 = \underline{a}_1^2, \underline{a}_1^2 = \underline{a}_1^3, \dots, \underline{a}_1^n = \underline{a}_1^{n+1}, \dots \}$$

and then construct a $\mu^3Q = (\Omega)$ with ν included in μ_1 in Ω .

We now construct formal systems which will ensure that case I and case II hold.

$$\underline{\mu}^1Q =$$

To ensure that case I holds, that is where there is but one domain of quantification for all worlds we construct system

$\mu^1Q =$. This system has the same primitive symbols as $\mu^4Q =$.

The formation rules will need to be altered so that 2 is replaced by 2'.

2'. If F^m is an m -ary predicate variable, and if $\underline{a}_1, \dots, \underline{a}_m$ are in individual constants (not necessarily distinct) then

$$F^m \underline{a}_1 \underline{a}_2 \dots \underline{a}_m$$

is a wff.

and 6 is replaced by 6'.

6'. If \underline{A} is a wff and \underline{x} is any individual variable then $(\underline{U}\underline{x})$

$(\underline{A} (\underline{x}/\underline{a}))$ is a wff, where $(\underline{A} (\underline{x}/\underline{a}))$ is the result of

substituting \underline{x} for \underline{a} at zero or more occurrences of \underline{a} in \underline{A} .

So the wffs of $\mu^3Q =$ are a subset of the formulae of $\mu^1Q +$.

A new set of conditions, C^1 , for membership of any μ_n in any $\mu^1Q = (\Omega)$ will omit from C^4 the rules $(C.\mu_n)$, $(C.DD)$, $(C.E_n)$ and $(C.U_n)$, (or omit from C^3 the rules $(C.E_n)$ and $(C.U_n)$), and include the rules

(C.E) If $(\underline{E}\underline{x})\underline{A} \in \mu_n$ then if \underline{b} occurs in any formula of any μ_m then $(\underline{A}(\underline{b}/\underline{x})) \in \mu_n$.

The rules in C^1 , together with the altered formation rules, ensure that any individual constant can occur in any kind of formula in any μ_n . So it is the one set of individuals whose members can be designated by individual constants in any μ_n . We could have simplified $\mu^1Q =$ by taking as individual constants for the system those with one superscript, say 1. But this would have made the relationship between the set of wffs of $\mu^1Q =$ and the sets of wffs of the other three systems far more complex.

$\mu^2Q =$

The system $\mu^2Q =$ would ensure that case II holds, and would have all the primitive symbols of $\mu^4Q =$ but with the addition of individual constants by the alteration, for individual constants only, of $(n \geq 1)$ to $(n \geq 0)$. The constants with the superscript 0 would be taken as designating individuals in the set of individuals common to all worlds. The formation rules would be the same as for $\mu^4Q =$. A new set of conditions, C^2 , will omit from C^4 the rules (C.E_n) and (C.DD), and add the rules:

(C.DD⁰) If \underline{a}_j^i and \underline{b}_m^k are individual constants, $(i \geq 0)(k \geq 0)$, in any member of a member of Ω and $i \neq k$ then for every

$$\mu_n \text{ in } \Omega \quad \underline{a}_j^i \neq \underline{b}_m^k \in \mu_n.$$

(C.E⁰) If $(\exists \underline{x})\underline{A} \in \mu_n$ then $(\underline{A}(\underline{a}^0/\underline{x}))$ or $(\underline{A}(\underline{b}^n/\underline{x})) \in \mu_n$ for at least one individual constant \underline{a}^0 or one individual constant \underline{b}^n .

(C.U⁰) If $(\forall \underline{x})\underline{A} \in \mu_n$ then if \underline{b}^0 occurs in the formulae of any μ_m then $(\underline{A}(\underline{b}^0/\underline{x})) \in \mu_n$.

Rule (C.DD) is replaced with (C.DD⁰) because the proviso in the former that $(j \geq 1)(i \geq 1)$ does not prevent identities of the form

$$\underline{a}_1^0 = \underline{b}_1^3$$

being in any μ_n . If such identities were to be allowed then by transitivity of identity we might get, from $\underline{a}_1^0 = \underline{b}_1^3$ and $\underline{a}_1^0 = \underline{b}_1^4$, $\underline{b}_1^3 = \underline{b}_1^4$.

The Existential and Universal instantiation rules have been expanded so that instantiations can be made in any μ_n to constants with a superscript which is either n or 0. Clearly the individual constants with the 0 superscript name the individuals in the set of individuals common to all the worlds. If it is true that ' \underline{a}^0 ' and ' \underline{b}^0 ' name the one individual in the set common to all worlds then this must be so in every world, according to our assumptions. The rule (C. $\mu =$) will ensure that if ' \underline{a}^0 ' and ' \underline{b}^0 ' name the one individual then ' $\underline{a}^0 = \underline{b}^0$ ' will appear in every μ_n in which either ' \underline{a}^0 ' or ' \underline{b}^0 ' occurs.

In constructing these four systems to reflect the four cases of designation and domains of quantification there are two important groups of rules. The first group of rules (C. μ_n), (C.DD), (C.DD⁰) and (C. $\mu =$) ensure that the sets of individuals in each world are in accordance with cases I to IV, and that the referent of any individual constant is uniform in a given model system. The rule (C. $\mu =$), which is in every system, ensures that constancy of reference, so that if ' \underline{a}^i ' and ' \underline{b}^j ' refer to one individual in one world then ' $\underline{a}^i = \underline{b}^j$ ' will appear in every μ_n in which any reference at all is made to either \underline{a}^i or \underline{b}^j . The other rules within this group are used to ensure that various sets of individuals are kept distinct where this is required, and the only reference made to individuals in other worlds with distinct sets of individuals is in trans-world identity formulae. The rules (C. μ_n) and (C.DD) occur in $\mu^4 Q =$ and ensure that the only occurrence of formulae in a μ_n

containing superscripts other than n are occurrences of trans-world identity formulae, and all transworld identities are false. A similar situation occurs in $\mu^2Q =$ but with allowance for a set of individuals common to all worlds.

The second group of rules which is important is the group of existential and universal instantiation rules. These rules indicate from system to system just what the domains of quantification are. It is clear from these rules that the domain of a quantifier in a μ_n is the set of individuals in the world of which μ_n is the (partial) description. We could say that quantifiers are world relative in every $\mu^iQ = (1 \leq i \leq 4)$.

It should also be noted that, in $\mu^1Q =$ and $\mu^3Q =$, identity and predication are treated somewhat differently, even though we have called '=' a 'predicate constant'. We have already pointed out that in all these systems naming is constant across all possible worlds, but predication (of other kinds) is not. For example, if ' \underline{a}^i ' designates the same individual as ' \underline{a}^j ', then ' $\underline{a}^i = \underline{a}^j$ '; and ' \underline{a}^i ' can be substituted for ' \underline{a}^j ' in all formulae, but the properties of \underline{a}^i can change from world to world in a quite arbitrary fashion. So we can have $F\underline{a}^i \in \mu_6$ and $F\underline{a}^j \in \mu_6$, but $\sim F\underline{a}^i \in \mu_7$ and $\sim F\underline{a}^j \in \mu_7$.

Although the difference in the treatments of identity and predication is quite clear in $\mu^1Q =$ and $\mu^3Q =$, there is a difference also in $\mu^2Q =$ and $\mu^4Q =$, even if that difference is simply that trans-world identities are given a separate status. These differences overall indicate that our individual constants are being treated as rigid designators as we presupposed they should be.

It is possible to formalize this rigid designation relation in the type theoretic system of Appendix I. When this is

done, as it is in the Appendix, it is clear that the relation ' x rigidly designates y ' is the same relation as ' x names y '. In the systems $\mu^i Q = (1 \leq i \leq 4)$ we have assumed the principle of rigid designation and have applied it in a kind of pre-modal logic.

We can treat membership of a μ_n as though the formulae which are members were true in world n . It is in that light that we can turn now to some of the consequences of our definition of self-sustainability. If a formula is self-sustaining it does not follow that it is true in all possible worlds under cases II and IV.

For example, in systems $\mu^4 Q =$ and $\mu^2 Q =$, the formula

$$(Ux)Fx \supset Fa^1$$

is self-sustaining. Its negation is not satisfiable in any μ_n . It is satisfiable only in μ_1 . The point is that this formula and its negation are both prevented, by $(C. \mu_n)$, from being a member of any μ_n where $n \neq 1$, because this formula contains a^1 . The negation of this formula cannot be a member of μ_1 . One way of expressing this fact is to say that in any maximal μ_1 , consistent with the rules in C^1 , $((Ux)Fx \supset Fa^1)$ will be a member, even though this formula is not satisfiable in any other μ_n .

In general, with the exception of trans-world identities, in $\mu^4 Q =$ and $\mu^2 Q =$ self-sustaining formulae containing individual constants with superscripts equal to or greater than 1 are not true in all possible worlds but are true only in one world. If, as is the case in $\mu^2 Q =$, a self-sustaining formula contains individual constants with the superscript 0, then such a formula will be true in all possible worlds. All self-sustaining formulae containing no individual constants will be true in all possible worlds.

In the other systems, $\mu^1Q =$ and $\mu^3Q =$, all self-sustaining formulae are true in all possible worlds. The world-relative self-sustainability of formulae which occurs in $\mu^2Q =$ and $\mu^4Q =$ does not occur in the other systems.

Identity and essential properties

When we discussed the time travel story two ways of avoiding an unwanted conclusion were suggested. In the first it was assumed that the time traveller had actually gone from one world, g , to another, f , and that he was the only member common to both. This assumption could be expressed in a model system of $\mu^3Q =$, or in a model system of $\mu^2Q =$ provided that there was either only one individual constant with 0 as superscript or only one individual designated by all constants of superscript 0. Neither $\mu^1Q =$ nor $\mu^4Q =$ could be used.

In the second way it was assumed that the worlds "branched". Again this could be expressed in a model system of either $\mu^2Q =$ or $\mu^3Q =$. In a way, $\mu^2Q =$ is more intuitively acceptable as a means of expressing this way of avoiding the unwanted conclusion because the system is constructed with a domain of individuals common to all possible worlds. Nevertheless we could use $\mu^3Q =$. But surprisingly enough, we could also use $\mu^1Q =$ for the description of this "branching" system of worlds. If we do use $\mu^1Q =$ then we are assuming that everyone in the actual world, g , is also in the time traveller's destination world, f , and that no one else is in the world f . (It serves our purposes for the moment to confine our attention to persons, but the same can be said of objects, places, etc.)

If we use $\mu^1 Q =$ then our assumptions are those set out in case I. Applying the assumptions of case I to the "branching" world notion means that one and the same set of persons is supposed to be common both to the trunk and one branch and also to the trunk and the other branch. This is easy enough to grasp as far as the trunk goes. This is the common history. But what does this amount to from the branching on?

Under the assumptions of case I we could say that the individuals who live before and through the branching will belong to the set designated by the constants superscripted by 1. But what of those who belong wholly to one of the branches? Let us suppose that, in our Punic story, in the branch f Hannibal has a son after the fall of Rome whom he calls "Napoleon". Are we to identify Napoleon in f with the Emperor Napoleon in g ? There are several responses which could be made to the question.

One response would be simply to say that natural language names like "Napoleon" are often ambiguous. In formalizing we must, because of our assumptions about the nature of names in an ideal language, compensate for these ambiguities by formalizing the same ordinary language name with more than one individual constant when the ordinary language name is ambiguous. This is somewhat like treating the word "Napoleon" as a means for the expression of more than one name, or of only a part of a full name. This makes the name "John Smith" something like the term "bank". Ordinary language makes do with as few words as possible.

But even if we accept the notion that names in ordinary language can be ambiguous, this does not really give the appropriate response to the question. The question demands an answer to the problem of how the individuals (persons) in f can be the same

individuals (persons) as those in g . Does it make sense to say that whereas Napoleon was born in the real world in Corsica in 1769, in the world of our story he was born in Rome in 217 B.C.? What are we to make of the difference in pedigree? And although we might go on to retail a story of what happened in f so that Napoleon becomes Emperor of all Europe and much of Africa and dies in his fifty-second year, there is not much to convince one that the two are really the one person, apart from our story in which we simply give the name "Napoleon" to the son of Hannibal. It is not that there is any contradiction, if this is done in $\mu^1 Q =$. We can have two descriptive sets, μ_1 and μ_2 , and include in μ_1 the formula " $Bn_1^6 \& Ln_1^6$ " to be read as "Napoleon was born in the year 1769 and lost the Battle of Waterloo", and include in μ_2 the formula " $\sim Bn_1^6 \& Sn_1^6$ " to be read as "Napoleon was not born in the year 1769 but was the son of Hannibal." Although the formulae are contradictory if in the one μ_n , in the system $\mu^1 Q =$ when they are not in the same μ_n there is no inconsistency.

Another response which can be made to the question of whether we should identify Napoleon in f with Napoleon in g is that we should do so if, and only if, there are some specially designated properties common to both. For example, it might be necessary for such an identification that both be male homo sapiens. This response is part of a basic theory that any individual has some essential properties which identify him (or it), and which he (or it) will have in every world in which he (or it) appears. Only the possession of such essential properties would licence the giving of a proper name to an individual in a second possible world when that name has already been given to someone in another possible world.

If we consider again our time-traveller we could describe the situation in the following terms: A man wakes up and finds

himself in an altogether strange world, apparently the Roman world of 220 B.C. He discovers perhaps that his face is different to what he remembers it to be. Yet he has what certainly seem to be coherent memories of the 21st Century and also of entering a time machine. It would certainly not be absurd for this man to reckon himself to be Wells and to be a time-traveller. The decisive factor for him would be whether he counted what appeared to be memories to be memories indeed. The point here is not that this man's beliefs about what seem to be memories will show that he is Wells, or is not Wells, but just that what appear to be memories could constitute a formidably persuasive influence whether or not he comes to believe that he was Wells. He can hardly believe that he does remember the 21st century without believing that he is a time-traveller. This follows from the truth that one can remember being x only if one is x.¹

Far more problematic is the Napoleon case. There is no question of memory there. We have two people, born in different circumstances, placed differently in history, dying in different circumstances. The notion that there might be properties, possessed by both and by no one else, which identify both as the bearer of the one name, and hence, by our theory and our assumptions, the one person, is rather a strange notion.

But, in order to explore this question of the identity of individuals from world to world we begin by introducing modal operators and extending the logics set out above. At first we will be concerned with the introduction of alethic modalities. These modalities will not be introduced just in order to consider this problem of cross-world identity, but also various other problems. In discussing cross-world identity and associated problems we will discuss the problem of essentialism. In particular, we will discuss

¹Cf. A. Flew, "Locke and the Problem of Personal Identity" in Locke and Buckley, eds. C. B. Martin and D. M. Armstrong, Macmillan, London especially pp. 159-164.

whether each individual has some property in all possible worlds, a property which identifies that individual from world to world.

Modal Systems

Let us begin by considering what happens when we add the primitive symbol ' \Box ' to the improper symbols of each $M^nQ = (1 \leq n \leq 4)$. And let us read ' $\Box A$ ' as 'In every possible world: A ', and ' $\Diamond A$ ' as 'In at least one possible world: A .' We call the extended systems $M^nQ = (1 \leq n \leq 4)$, and assume for the moment that if A is a formula so are $\Box A$ and $\Diamond A$.

We need to add a formation rule in each case, but there is no need to alter the membership of Ω . But there will have to be extensions to the consistency rules in each case. Before setting out new rules and sets of rules for each $M^nQ = (1 \leq n \leq 4)$ we will discuss what follows from the suggested readings of the modal operators, and some of the problems which arise. We assume for the moment that each $C^i (1 \leq i \leq 4)$ is included in the respective extended set of conditions.

It ought to follow from the suggested reading of ' $\Box A$ ' that if A is a tautology then $\Box A$ will be self-sustaining, since A will be the case in every possible world. Similarly, if B is a theorem of predicate calculus in which there are no individual constants, then $\Box B$ will be self-sustaining. And if we were to construct maximal model sets, then we would expect every tautology and every theorem of predicate calculus in which there are no individual constants to be members of each maximal model set. In these cases it follows from the reading of ' $\Box A$ ' that $\Box A$ will be true in every possible world, and so $\Box \Box A$, and $\Box \Box \Box A$, etc. Also in these cases,

if $\Diamond \Box A$ is self-sustaining then $\Box A$ is in at least one world, and therefore $\Box A$ is self-sustaining.

Prima facie, for tautologies and theorems of predicate calculus without individual constants, the modality indicated for each $M^iQ = (1 \leq i \leq 4)$ is S5. The question is, what happens to formulae in which there are constants? For they are all indexed by their superscripts as belonging, in some sense, to one world, except in the case of identities. An identity such as ' $\underline{a}^1 = \underline{b}^5$ ', can hardly be said to belong to one and only one world.

Identity and Modality

In discussing what happens to formulae containing individual constants we will turn our attention first to identities. We will discuss formulae in which no quantifiers appear and in which all the atomic parts of the formulae are identities. We will discuss formulae in which quantifiers appear later.

Turning first to identities we notice that, since in $M^1Q =$ the one set of individuals is common to all possible worlds, if $\underline{a}^7 = \underline{a}^6$ is true in any one world then it will be true in all worlds. So if $\underline{a}^7 = \underline{a}^6 \in \mu_n$ for some n , then because of rule (C. $\mu =$) we would expect $(\underline{a}^7 = \underline{a}^6)$ to be satisfiable in every μ_n . Similarly if $\underline{a}^5 \neq \underline{a}^4 \in \mu_n$ for some n then we would expect $(\underline{a}^5 \neq \underline{a}^4)$ to be satisfiable in every μ_n .

In formal terms this would mean that in $u^1Q =$,

$$(1) \quad \underline{a}^i = \underline{b}^j \equiv \Box (\underline{a}^i = \underline{b}^j)$$

and
$$(2) \quad \underline{a}^i \neq \underline{b}^j \equiv \Box (\underline{a}^i \neq \underline{b}^j)$$

and, by definition,

$$(2') \quad \underline{a}^i = \underline{b}^j \equiv \Diamond (\underline{a}^i = \underline{b}^j)$$

would all be self-sustaining. Also the formulae

$$(3) \quad \underline{a} = \underline{a} \equiv \Box(\underline{a} = \underline{a})$$

would be self-sustaining. In one sense this means that modal operators are vacuous when their scopes are atomic identities, but in another sense it is trivially true that all identities are true in all possible worlds.

In $M^3Q =$ where it is possible to have a set of individuals common to all possible worlds it is also understandable that (1), (2), and (3) should be self-sustaining. In $M^2Q =$ and $M^3Q =$ (1) will be self-sustaining simply because both sides of the equivalence will always be false when $i \neq j$, and if $i = j$ then if $\underline{a}^i = \underline{b}^j$ is true then $\underline{a}^i = \underline{b}^j$ will be in every μ_n because of rule (C. $\mu =$). In $M^2Q =$ and $M^3Q =$ (2) will be self-sustaining because if $i \neq j$ then $\underline{a}^i \neq \underline{b}^j$ will be in every μ_n and both sides of the equivalence true. If $i = j$ then if $\underline{a}^i = \underline{b}^j$ is false it will be false in every μ_n because of rule (C. $\mu =$). In $M^2Q =$ and $M^4Q =$ (3) is clearly self-sustaining. So (1), (2) and (3) are self-sustaining in all systems, and the self-sustainability of these formulae in all systems simply re-inforces the notion that individual constants are rigid designators in these systems.

Our interpretation of the modal operator ' \Box ' as 'In all possible worlds' does raise some problems at this point. These problems have an important bearing on what we shall say later about quantifying into modal contexts, so we must discuss them. Consider reading

$$(4) \quad \Box (\underline{a}^i = \underline{b}^j)$$

as

$$(5) \quad \text{In all possible worlds } \underline{a}^i \text{ names what also } \underline{b}^j \text{ names.}$$

The problem is that (5) can be taken as saying that ' \underline{a}^i ' is a name

in each possible world of something and ' \underline{b}^j ' is a name in each possible world of the same thing. Both names occur in all possible worlds as the names of some one thing. This account of what (4) means is not problematic in $M^1Q =$ and $M^3Q =$ where ' \underline{b}^j ' can be used in μ_i when $i \neq j$ to describe something.

$F\underline{b}^j \in \mu_i$ is not ruled out by any member of either C^1 or C^3 . So, if an individual constant, \underline{a}^i , occurs in a μ_n ($n \neq i$) in a $M^1Q =$ or a $M^3Q =$ model system we could assume that ' \underline{a}^i ' is used as a name in the world (partially) described by μ_n .

But when we turn to $M^2Q =$ and $M^4Q =$ to consider this account, there are problems. What does it mean to say that ' \underline{a}^i ' and ' \underline{b}^j ' occur in μ_n ($(i \neq j)(i \neq n)(j \neq n)(i \geq 1)(j \geq 1)$) as names of something? μ_n is the (partial) description of a world in which there is nothing named by either ' \underline{a}^i ' or ' \underline{b}^j '. What ' \underline{a}^i ' names is in world i , and what ' \underline{b}^j ' names is in world j . (4) is false in $M^2Q =$ and $M^4Q = (i \neq j)$ just because what one name designates cannot be what the other designates.

The problem can be solved by one of two moves. The first is to reject the reading of (4) as (5). The second is to accept that if an individual constant occurs in a μ_n then, in the world (partially) described by μ_n that individual constant is accepted as a name.

If we don't accept the reading of (4) given in (5), and also reject the second alternative just suggested, then this places model sets in a strange position. Individual constants are being used in them to (partially) describe worlds without any guarantee that those constants are taken to be names in the worlds described. And problematic as the status of transworld identities is in any μ_n , the individual constants occurring in them have to

be accepted as names. Even in the world (partially) described we can conceive of someone saying, as they might in ours, "Pickwick is not identical with anyone in our world."

We will accept the second alternative above. Just as "Pickwick" is accepted as a name in our world so we will assume that ' \underline{a}^i ' ($i \neq n$) is acceptable in the world described by μ_n as a name. This assumption also means that $(E\underline{x})(\underline{x} = \underline{a}^i)$ in

$$(6) \quad (E\underline{x})(\underline{x} = \underline{a}^i) \in \mu_n$$

means not only

Something is \underline{a}^i (in world n)

but that Something is called ' \underline{a}^i ' (in world n).

So it is no wonder that $(E\underline{x})(\underline{x} = \underline{a}^i)$ is satisfiable in μ_n ($n \neq i$) in $M^1Q =$ and $M^3Q =$ but is not satisfiable in $M^4Q =$, or in $M^2Q =$ when ($i \geq 1$), especially since we have already indicated that the domain of quantification is always the set of individuals occurring in the world described by the μ_n . And if we remember that a μ_n does not have to be a complete novel then we can have the situation where some names are not mentioned. Only those mentioned are clearly to be taken as accepted in the world described.

Quantification and Modality

Once we introduce, as we have in (6), formulae in which there is a combination of modal operators, quantifiers and individual constants a series of problems arises. It is not very problematic to have closed formulae as the scopes of modal operators,¹ but there are three other forms of formulae which give rise to problems.

¹If $\Box \underline{A}$ is a formula then \underline{A} is the scope of \Box , and if $\Box \underline{A}$ is the scope of a quantification then \underline{A} is the scope of \Box .

So far we have indicated that the quantifiers in the systems $\mu^i Q = (1 \leq i \leq 4)$ have as their domains those individuals that occur in the world for which the μ_n in which the quantifier occurs provides a description. We have given an intended sense for the modal operators. We now consider what is to be made of a formula which consists of an open formula as the scope of an operator, or of a formula in which there is a variable bound in the scope of the modal operator by a quantification not in the scope of the modal operator.

Let us consider first formulae of the form $\underline{Q} \underline{Q} \underline{A} (\underline{a}^i)$ where \underline{Q} is either \Box or \Diamond , \underline{Q} is either (Ux) or (Ex) , and $\underline{A}(\underline{a}^i)$ is the scope of \underline{Q} and contains at least one individual constant with the superscript i ($i \geq 0$). For the moment it will simplify the discussion if we insist that $\underline{A}(\underline{a}^i)$ does not contain a trans-world identity.

If \underline{Q} , in $\underline{Q} \underline{Q} \underline{A} (\underline{a}^i)$, is \Diamond then there should be no real problem. We then give the general reading:

For at least one possible world: $\underline{Q} \underline{A} (\underline{a}^i)$.

Because of the formation rules for $M^4 Q =$, $M^3 Q =$ and $M^2 Q =$, especially 6, all the individual constants in \underline{A} will be \underline{a}^i . In system $M^4 Q =$, unless we alter $(C. \mu_n)$ both $\Diamond \underline{Q} \underline{A} (\underline{a}^i)$ and $\underline{Q} \underline{A} (\underline{a}^i)$ will be satisfiable but only in μ_i and only if $\underline{Q} \underline{A} (\underline{a}^i)$ is not itself contradictory. We could alter $(C. \mu_n)$ to

$(C. \mu_n^\Diamond)$ If \underline{A} contains any individual constant \underline{a}^i occurring outside the scope of a modal operator and outside of an identity and $i \neq n$, then $\underline{A} \notin \mu_n$.

If $(C. \mu_n)$ is replaced by $(C. \mu_n^\Diamond)$ for $M^4 Q =$ then $\Diamond \underline{Q} \underline{A} (\underline{a}^i)$ would be satisfied in any μ_n provided $\underline{Q} \underline{A} (\underline{a}^i)$ was satisfiable in μ_i .

The situation is similar in system $M^2Q =$ to that in $M^4Q =$, but with one important difference. In $M^2Q =$, if (\underline{a}^i) has the superscript 0 then $\Diamond \underline{Q} \underline{A} (\underline{a}^i)$ and $\underline{Q} \underline{A} (\underline{a}^i)$ both fall outside the scope of (C, μ_n) (and (C, μ_n^\Diamond)). So if $\underline{Q} \underline{A} (\underline{a}^0)$ is satisfiable in a μ_n we could assume that $\Diamond \underline{Q} \underline{A} (\underline{a}^0)$ would be satisfiable in any μ_n . Otherwise, for $i \geq 1$, the situation is the same as for $M^4Q =$.

The situation in $M^3Q =$ is the same for all formulae of the forms $\Diamond \underline{Q} \underline{A} (\underline{a}^i)$ and $\underline{Q} \underline{A} (\underline{a}^i)$ as it is in $M^2Q =$ for those of the forms $\Diamond \underline{Q} \underline{A} (\underline{a}^0)$ and $\underline{Q} \underline{A} (\underline{a}^0)$. The rule (C, μ_n) is not a member of C^3 , so $\underline{Q} \underline{A} (\underline{a}^i)$ is not rendered not satisfiable in μ_n just because $i \neq n$.

In system $M^1Q =$, although it is unlike the other three in that there can be a mixture of superscripts in the scope of a quantification, the situation is as in $M^3Q =$.

So, we would expect, on these considerations alone, that each system would have the same consistency rule, namely

(C. \Diamond^*) If $\Diamond \underline{A} \in \mu_n \in \Omega$ then there is in Ω at least one model set, such as μ_m , such that $\underline{A} \in \mu_m$ and μ_m is an alternative to μ_n in Ω .

One thing should be noted about rule (C. \Diamond^*) at this point. It concerns the phrase "alternative to". The phrase is part of the technical phrase "alternative to μ_n in Ω " which is defined in the rule. There is nothing in rule (C. \Diamond^*) which prevents μ_n being an alternative to itself in Ω . The relation between a set and its alternative, set up by rule (C. \Diamond^*), is called by Hintikka "the alternativeness relation", and can be reflexive.¹ Since we are using Hintikka style semantics we fall in with his terminology. At the same time, though the alternativeness relation can be reflexive, it

¹J. Hintikka, Knowledge and Belief. Cornell, Ithaca, 1962, p. 42.

will be convenient for the sake of the sorts of proofs used in these systems to assume that generally μ_m is not μ_n when μ_m is an alternative to μ_n in Ω .

We now consider what the situation will be if $\Box \Box A(a^i)$ is \Box . We have already suggested that (C, μ_n) in C^4 and C^2 be altered to (C, μ_n^\diamond) . If this suggestion is followed then $\Box \Box A(a^i)$ ($i \geq 1$) is not rendered not satisfiable in μ_n in $M^4Q =$ and $M^2Q =$, simply because $i \neq n$. Nevertheless, since $\Box A(a^i)$ is not satisfiable in $M^4Q =$ and $M^2Q =$ when $i \geq 1$ and $i \neq n$ we can hardly agree that:

In all possible worlds: $\Box A(a^i)$ is satisfiable in $M^4Q =$ and $M^2Q =$. So, we will have the situation where, in $M^4Q =$ and $M^2Q =$, all formulae of the form $\Box \Box A(a^i)$ ($i \geq 1$), are not satisfiable. It is this case which requires the additional clause in the definition of self-sustaining formulae that a formula is self-sustaining if the unit set of its negation is not satisfiable and the formula is satisfiable. Without this last clause in the definition, since every formula of the form $\Box \Box A(a^i)$ is not satisfiable, it would follow that $\sim \Box \Box A(a^i)$ was self-sustaining. From this it would follow that $\Diamond \sim (Ux)(Fa^i \supset Fa^i)$ was self-sustaining, and hence that $\Diamond (Ex)(Fa^i \& \sim Fa^i)$, and this last formula reads as:

In at least one possible world: a^i is both F and not F. And it will just not do to have such a proposition as a logical truth.

So, in $M^4Q =$ and $M^2Q =$ formulae of the form $\Box \Box A(a^i)$ ($i \geq 1$) are going to be neither satisfiable nor self-sustaining.

On the other hand in the systems $M^1Q =$ and $M^3Q =$ it will be possible for formulae of the form $\Box \Box A(a^i)$ to be both satisfiable and self-sustaining. Also, in $M^2Q =$ it will be possible for formulae of the form $\Box \Box A(a^0)$ to be both satisfiable and self-sustaining.¹

¹Examples are given in the Appendix.

In the system $M^1Q =$ we have the additional feature that in $A(a^i)$ there could be individual constants other than those with the superscript i . But this does not bring about any real point of interest, unless we consider quantifications containing in their scopes trans-world identities. We now take up that consideration. It is only in $M^1Q =$ where there can be quantifications with trans-world identities in their scopes. In the other systems these are ruled out by the formation rules.

We need to consider formulae with both modal operators.

Let us take the examples

$$(a) \quad \Diamond (Ex)(x = a^1 \ \& \ a^1 = b^2)$$

and

$$(b) \quad \Box (Ex)(x = a^1 \ \& \ a^1 = b^2).$$

(a) can be read as:

(a*) In at least one possible world something is a^1 and a^1 is b^2 .

(b) can be read as:

(b*) In every possible world something is a^1 and a^1 is b^2 .

If we take (a*) to mean that

$$(Ex)(x = a^1 \ \& \ a^1 = b^2) \in \mu_n \text{ for some } n$$

then it follows that

$$b^1 = a^1 \ \& \ a^1 = b^2 \quad \text{for some } b^1$$

We have already seen that in $M^1Q =$ every identity is equivalent to its necessitation. So it will follow that (a) is equivalent to (b).

Quantifying into Modal Contexts

We now turn to a consideration of the second set of formulae in which modal operators and quantifiers occur. These are formulae of the form $Q \ Q \ A$ where there are no individual constants in

A. It can be argued that in $M^4Q =$ and in $M^2Q =$ we should not permit the construction of formulae of the form $Q \subseteq A(\underline{x}/\underline{a}^i)$, where $i \geq 1$ and \underline{x} is bound in Q , and the formulae contain no individual constants.

The argument against permitting the construction of such formulae is that the most reasonable interpretation of such formulae means that the modal operator is either unnecessary or makes the formulae contradict the underlying assumptions about the domains of quantification. For example

$$(U\underline{x}) \sqsubseteq F\underline{x} \in \mu_1$$

would be taken as indicating that each individual, \underline{x} , in the world described by μ_1 was such that in every possible world \underline{x} was F . Not only is everything in the world described by μ_1 to be F , but everything in the world described by μ_1 is to be F in every other possible world. From this we should be able to infer that everything in the world described by μ_1 is common to all the other possible worlds. This runs counter to the presuppositions of both $M^4Q =$ and $M^2Q =$. If on the other hand we take the example

$$(U\underline{x}) \diamond F\underline{x} \in \mu_1,$$

this would be taken as indicating that each individual, \underline{x} , in the world described by μ_1 was such that in at least one possible world \underline{x} was F . Obviously that one world would have to be the one described by μ_1 , for the reasons given above. So the \diamond is unnecessary, or vacuous.

If we do permit the construction of formulae such as $(U\underline{x}) \sqsubseteq F\underline{x}$ and $(U\underline{x}) \diamond F\underline{x}$, then those of the first kind will be neither satisfiable nor self-sustaining, they will just not ever be members of any model set in $M^4Q =$ or $M^2Q =$. Those of the second kind will be at least candidates for satisfiability or self-sustainability, but

the \diamond will be vacuous.

Against this argument it must be pointed out that there are exceptional cases in $M^2Q =$. For example, if we have an existential quantifier, then there are not the same problems. The formula

$$(E_x) \square Fx \in \mu_1$$

in system $M^2Q =$ would make good sense when taken as indicating that at least one of the individuals in the world (partially) described by μ_1 , namely one of those designated by an individual constant with a 0 superscript, say a^0 , is in all possible worlds E .

This rejoinder does not remove the bar to formulae of the type under discussion in $M^4Q =$. We can rule out formulae of this type in $M^4Q =$ by revising the formation rule 6. to

6M: If A is a wff and x is any individual variable, then provided that all the individual constants in A have the same superscript and provided that no modal operators occur in A , then $(Ux)(A(x/a))$ is a wff, when $(A(x/a))$ is the result of substituting x for a at zero or more occurrences of a in A .

We also need the rule:

7. If A is a wff then so is $\square A$.

Because of the exceptions in $M^2Q =$ it is best simply to add 7 to the formation rules, and let the other problems work themselves out in the ways indicated. The readings we have suggested for the various combinations of quantifications and modal operators do not raise problems in $M^3Q =$ and $M^1Q =$.

Finally we consider the third set of formulae where formulae are of the form $Q Q A (a^i)$ and a^i occurs at least once in A . If we accept 6 M instead of 6 in the formation rules for $M^4Q =$ then there is no problem in $M^4Q =$ because there are no formulae of the form

under discussion.

The situation in $M^2Q =$, which will be of interest, is where the form under discussion is

$$(E\underline{x}) \sqcup \underline{A} (\underline{a}^0).$$

In particular the two formulae

$$(7) \quad (E\underline{x}) \sqcap (\underline{x} = \underline{a}^0)$$

and

$$(8) \quad (E\underline{x}) \diamond (\underline{x} = \underline{a}^0)$$

are of interest.

If (7) is read as

(7') There is something which, in all possible worlds, is \underline{a}^0 then this seems to be trivially true in $M^2Q =$. A parallel reading of (8) would also seem to be trivially true.

Given that (7) is the member of some μ_n , then we can read (7) in

$$(7'') \quad (E\underline{x}) \sqcap (\underline{x} = \underline{a}^0) \in \mu_1$$

as indicating that there is something in world 1 which is, in all possible worlds, named ' \underline{a}^0 '. Similarly we can read

$$(8'') \quad (E\underline{x}) \diamond (\underline{x} = \underline{a}^0) \in \mu_1$$

as indicating that there is something in world 1 which is, in at least one possible world, named ' \underline{a}^0 '. Reading (7'') and (8'') in this way is simply to apply the interpretation of (6) above. Of course this does not alter the point that in $M^2Q =$ (7) and (8) are trivially true, and they are self-sustaining in $M^2Q =$.

We now consider the status of

$$(7') \quad (E\underline{x}) \sqcap (\underline{x} = \underline{a}^i)$$

and

$$(8') \quad (E\underline{x}) \diamond (\underline{x} = \underline{a}^i)$$

in the other three systems. It should be clear that both of these

will be self-sustaining in $M^1Q =$. In $M^3Q = (7')$ is not self-sustaining. As the member of a model set $(7')$ indicates that something in the world described, namely \underline{a}^1 , is in every possible world. That, in $M^3Q =$, is not a logical truth. Also in $M^3Q = (8')$ is not self-sustaining. As the member of a model set $(8')$ indicates that something in the world described, namely \underline{a}^1 , is in at least one possible world.

Neither $(7')$ nor $(8')$ are formulae of $M^4Q =$ because of the new formation rule 6 M. But they are both formulae of $M^2Q =$ when $(i \geq 1)$, and both are logically false. Their negations are self-sustaining, as one might well expect.

But we need to consider the formulae of $M^1Q =$ and $M^3Q =$ of the form

$$(U\underline{x}) \underline{Q} \underline{A} (\underline{a}^1)$$

Let us consider the situation in $M^3Q =$ first. Once again let us resort to particular cases:

$$(9) \quad (U\underline{x}) \sqcap (\underline{x} = \underline{a}^1) \in \mu_1$$

and

$$(10) \quad (U\underline{x}) \diamond (\underline{x} = \underline{a}^1) \in \mu_1.$$

If we read the formulae without taking cognisance of the model set of which each is a member then we get respectively

$$(9^*) \quad \text{Everything is such that in all possible worlds it is } \underline{a}^1$$

and

$$(10^*) \quad \text{Everything is such that in at least one possible world it is } \underline{a}^1.$$

The immediate problem is that it's not clear what the domain of quantification is. But if we consider the formulae with respect to some one μ_n then the quantifier can be made relative to the world (partially) described by μ_n . So we can read (9) and (10) as

- (9") Everything in world 1 is such that in all possible worlds it is \underline{a}^1 .

and

- (10") Everything in world 1 is such that in at least one possible world it is \underline{a}^1 .

In other words, world 1 contains only one individual \underline{a}^1 , and (9) indicates that that one individual is called ' \underline{a}^1 ' in all possible worlds while (10) indicates that that one individual is called ' \underline{a}^1 ' in at least one possible world.

The situation in M^1Q is simpler because we have the same domain of quantification for all worlds. So (9) indicates that that domain is just one item, \underline{a}^1 , and so also does (10).

We now set out the rules for the membership of model sets as far as the rules affect modal operators.

- (C. \Diamond^*) If $\Diamond \underline{A} \in \mu_n \in \Omega$ then there is in Ω at least one model set, such as μ_m , such that $\underline{A} \in \mu_m$ and μ_m is an alternative to μ_n in Ω .

- (C. \Box) If $\Box \underline{A} \in \mu_n \in \Omega$ then $\underline{A} \in \mu_n$.

- (C. $\Box\Box^*$) If $\Box \underline{A} \in \mu_n \in \Omega$ and μ_m is an alternative to μ_n in Ω then $\Box \underline{A} \in \mu_m$.

- (C. $\Box\Box^*$) If $\Box \underline{A} \in \mu_m \in \Omega$ and μ_m is an alternative to μ_n in Ω then $\Box \underline{A} \in \mu_n$.

We have already noted one thing about the rule (C. \Diamond^*)¹. That concerned the phrase "is an alternative to". There is a second point to which we now give attention.

The second point is more complex and concerns the system M^3Q . Take the set

¹Supra p. 69.

$$\{ (E_X) \Box F_X, \sim \Box (E_X) F_X \}$$

To test this set for satisfiability we will see if it can be included in some μ_n . By the definition of \Diamond and $(C. \sim \sim)$, the second formula gives the assumption:

$$1. \quad \Diamond \sim (E_X) F_X \in \mu_n$$

From the other formula we get

$$2. \quad \Box F_{a_1}^n \in \mu_n \text{ by } (C.E_n), \text{ and then}$$

$$3. \quad F_{a_1}^n \in \mu_n \text{ by } (C. \Box)$$

from 1: $4. \quad \sim (E_X) F_X \in \mu_m \text{ by } (C. \Diamond^*), \text{ where } \mu_m$

is an alternative to μ_n in Ω . We could count μ_m as μ_n , but then we would get

$$5. \quad (U_X) \sim F_X \in \mu_n \text{ by the definition of } E \text{ and } (C. \sim \sim), \text{ and so we get}$$

$$6. \quad \sim F_{a_1}^n \in \mu_n$$

which is contradictory, because of 3. So we are forced to consider the possibility that the alternative to μ_n in Ω is not μ_n itself but some other arbitrary set. This gives

$$5'. \quad (U_X) \sim F_X \in \mu_m$$

and also $6'. \quad F_{a_1}^n \in \mu_m \text{ from 2 by } (C. \Box \Box^*) \text{ and } (C. \Box).$

Unless we change the rule of universal instantiation, $(C.U_n)$, to allow $(U_X) \sim F_X$ in μ_m to be instantiated to $F_{a_1}^n$, we will get no contradiction between 6' and the instantiation of 5'. There is also the possibility of leaving $(C.U_n)$ as it is, but of adding a rule to enable 5' and 6' to be shown to be contradictory. In order to keep the non-modal rules the same as before, we introduce the rule:

$(C. \Box \Box i)$ If $\Box A \in \mu_n \in \Omega$ and μ_m is an alternative to μ_n

in Ω or μ_n is an alternative to μ_m in Ω , then if

A contains occurrences of the individual constants $a_{i_1}^n, \dots, a_{j_k}^n$ then for some individual constants $b_{x_1}^m, \dots, b_{y_k}^m$

$$a_{i_1}^n = b_{x_1}^m \in \mu_m, \dots, a_{i_k}^n = b_{y_k}^m \in \mu_m.$$

From this rule, in our example above, we get

$$7'. \quad a_1^n = b_1^m \in \mu_m,$$

and by the substitutivity of identicals, we get

$$8'. \quad Fb_1^m \in \mu_m$$

and by (C.U_n) we get

$$9'. \quad \sim Fb_1^m \in \mu_m$$

and so there is a contradiction. So the set we began with is rendered not-satisfiable by the addition of rule (C. $\Box \Box i$).

Before adding (C. $\Box \Box i$) to the rules for $M^3Q =$ we must ask whether it is appropriate, in the light of our presuppositions and case III, that the set

$$\{ (E\bar{x}) \Box F\bar{x}, \sim \Box (E\bar{x})F\bar{x} \}$$

be inconsistent. It does seem so. The first formula is read as

"Something from world \bar{n} is, in all worlds, F"

The second is read as

"It is not the case that in all worlds something is F."

On these readings, if the first is true then the second should be false.

The question may then be asked, why not insert in the rules for $M^3Q =$ a rule analogous to (C. $\Box \Box i$) but for the operator \Diamond . In this case, such an insertion would render certain formulae self-sustaining when our readings indicate that they should not be so. We return to this in a moment, but before doing so we set out the proposed sets of conditions for the systems $M^iQ = (1 \leq i \leq 4)$ and some facts about formulae which are self-sustaining.

The set C_M^1 for $M^1Q =$ consists of C^1 together with the first four rules above.

The set C_M^2 for $M^2Q =$ consists of C^2 less (C. μ_n), and then added are

the first four rules above and rule (C, μ_n^\diamond) .

The set C_M^3 for $M^3Q =$, so far, consists of C^3 together with the five rules above.

The set C_M^4 for $M^4Q =$ consists of C^4 less (C, μ_n) and then added are the first four rules above and rule (C, μ_n^\diamond) .

Given these rules only we can turn attention to formulae like the Barcan formula and its converse. There are other formulae which show the commutation of quantifiers and modal operators. We will see which of these is self-sustaining in $M^1Q =$ and $M^3Q =$.

The Barcan Formula and others

The Barcan formula and its converse are self-sustaining in $M^1Q =: ((\underline{Ux})\Box Fx \equiv \Box (\underline{Ux})Fx)$. This accords well with our readings.

Everything in every world is F iff in every world everything is F.

We have to remember that in this case it is the one set of individuals which is in every world. Although $(\underline{Ex})\Box Fx \supset \Box (\underline{Ex})Fx$ is self-sustaining in $M^1Q =$, the converse is not. This also accords with our readings.

If at least one thing is such that in every world it is F, then in every world at least one thing is F.

is clearly self-sustaining, but not

If in every world at least one thing is F, then at least one thing is such that in every world it is F.

The counter example to the latter is where at least one thing is F in each world, but it is a different thing in each world, so no one thing is F across all worlds.

Also, in $M^3Q =$ the Barcan formula is not self-sustaining, nor is its converse. This accords with our readings so long as we

remember that the set of individuals in each world can be a different set. The Barcan formula then reads as:

If everything in every world is F then everything from world \underline{n} is F in every world.

The formula itself is falsified by the situation where world \underline{m} includes not only all the individuals from \underline{n} (which are F) but other individuals which are not F. The converse is falsified by the situation where not everything from \underline{n} is in every other world.

Also in $M^3_Q =$ the formula

$$(E_{\underline{x}}) \Box F_{\underline{x}} \supset \Box (E_{\underline{x}}) F_{\underline{x}}$$

is self-sustaining, but its converse is not. This also accords well with our readings. The formula reads as:

If something from world \underline{n} is F in all worlds then in all worlds something is F.

The converse reads as:

If in all worlds something is F then something from world \underline{n} is F in all worlds.

A counterexample to the converse is given by there being no intersection of domains of quantification. In each world something is F, but it is not the same thing which is F from world to world.

We now take up the discussion from the previous section about a rule for \Diamond analogous to (C. $\Box \Box i$) for $M^3_Q =$. Let us first set out such a rule.

(C. $\Diamond \Diamond i$) If $\Diamond \underline{A} \in \mu_{\underline{n}} \in \Omega$ then there is in Ω at least one model set, such as $\mu_{\underline{m}}$, such that $\underline{A} \in \mu_{\underline{m}}$ and if \underline{A} contains occurrences of the individual constants $\underline{a}_1^n, \dots, \underline{a}_{jk}^n$ then for some individual constants $\underline{b}_{x_1}^m, \dots, \underline{b}_{y_k}^m$ $\underline{a}_{i_1}^n = \underline{b}_{x_1}^m \in \mu_{\underline{m}}, \dots, \underline{a}_{j_k}^n = \underline{b}_{y_k}^m \in \mu_{\underline{m}}$.

The addition of this rule to C_M^3 will not affect the deduction which

shows the Barcan formula to be not self-sustaining. This can be checked in Appendix II. But $(C, \Diamond \Diamond i)$ will render the converse of the Barcan formula self-sustaining. The rule $(C, \Diamond \Diamond i)$ will not render the formula

$$\Box (Ex)Fx \supset (Ex) \Box Fx$$

self-sustaining.

Since we are trying to construct these systems to accord with our presuppositions and preferred readings we have one good reason for rejecting the introduction of $(C, \Diamond \Diamond i)$ into the set of rules for $M^3Q =$. The "mechanics" of system $M^3Q =$ are such that rule $(C, \Diamond \Diamond i)$ should not be introduced.

None the less, it would be best if we could give some other sort of reason as to why $(C, \Box \Box i)$ is desirable and $(C, \Diamond \Diamond i)$ is not, that is, a reason which does not rely simply on the production of proofs, some of which have welcome results because $(C, \Diamond \Diamond i)$ is not used and others of which have unwelcome results when $(C, \Diamond \Diamond i)$ is used. Since one formula which $(C, \Diamond \Diamond i)$ renders self-sustaining is the converse of the Barcan formula we turn to it.

The converse of the Barcan formula reads as:

If everything in every world is F then everything
from world n is F in every world.

The negation of this is satisfiable when not everything from world n is in every world, let alone being F in every world. When we formalize, in $M^3Q =$, the proposition that

It is not the case that everything in world n is F in
every world

we get

$$\sim (Ux) \Box Fx$$

By definition this gives us

$$(E\bar{x}) \Diamond \sim F\bar{x} \quad ,$$

which reads as

Something in world \underline{n} is, in at least one possible world,
not F.

And this reading is ambiguous. It could mean that something from world \underline{n} is in at least one possible world other than \underline{n} and there it is not F. But it could also mean that something in world \underline{n} is in at least one possible world -- namely \underline{n} itself -- and there is not F. And it could also mean that something in world \underline{n} is just not in any other world.

Our counterexample fits with the last two interpretations of this ambiguous reading, but not with the first. It is the first reading which would justify the introduction of $(C. \Diamond \Diamond i)$, but not the last two. Since our counterexample also fits with the case, case III, which $M^3Q =$ is supposed to reflect we reject rule $(C. \Diamond \Diamond i)$.

The Preferred Systems.

We have pointed out that model systems can be constructed in $\mu^3Q =$ to reflect each of the cases I to IV. So, in the pre-modal predicate logics we constructed, we could say that $\mu^3Q =$ is the most general system. The addition of modal operators does not change this in any basic way. So we will discuss most problems in terms of $M^3Q =$. The system $M^1Q =$ is also of interest simply because the one set of individuals occurs in all possible worlds.

So, in what follows we shall treat $M^1Q =$ and $M^3Q =$ as the preferred systems, the latter as the most preferable.

The indiscernibility of identicals

Let us now consider the principle of the indiscernibility of identicals relative to the two preferred systems. There are two formulae of interest. They are

$$(i) \quad (U_x)(U_y) \overline{\overline{(x = y) \& F_x \supset F_y}} \quad \text{and}$$

$$(ii) \quad (U_x)(U_y) \overline{\overline{x = y \supset \Box (x = y)}}$$

(i) is the well known law of substitutivity of identity for bound variables. The other is simply saying that identity is true for all possible worlds. Both (i) and (ii) are self-sustaining in $M^1Q =$ and $N^3Q =$.¹

Formula (ii) accords with what we have already said about identity and rigid designation. We have given identity a peculiar status in our systems, a status denied any other predicate. This is especially clear in the allowing of identities into sets even when the individual constants in the identities do not designate any individual in the world described by that model set. It is therefore important to note that although (ii) is self-sustaining, the following formula is not:

$$(iii) \quad (U_x)(U_y)(F_{xy} \supset \Box F_{xy})$$

A countermodel for (iii)² provides for F truly relating two things in one world, but not in another. This is to be contrasted with identity, where if ' $a = b$ ' is true in one world it will be so in all others.

Quine maintains that if we accept (ii) then we are committed to "Aristotelian essentialism" which holds that "some at least of the traits that determine an object do so necessarily."³ And it must be admitted that in all our logical systems we have

¹See Appendix II

²See Appendix II

³W.V. Quine, From a logical point of view

we have assumed that there is a fixed, determinate relation between individual constants and the objects they designate. But being named by a certain name can hardly be held to be a trait which determines an object. If it is, then Quine's suggestion that we use naming predicates to eliminate singular terms makes Quine a supporter of Aristotelian essentialism. If it is not, then it is hard to see just how (ii) can count in favour of what Quine says it counts in favour. Anyway, we will return to the question of essentialism in a later section.

Rigid designators and definite descriptions

In his paper, "Existential and Uniqueness Presuppositions",¹ Hintikka sets about constructing a quantified modal logic in which singular terms refer uniquely. By "singular terms" Hintikka means both proper names and definite descriptions, such as 'the morning star' or 'the next President of the United States'. Not only does he want both kinds of singular terms to refer uniquely, but he also adopts the policy of formalizing both names and definite descriptions with individual constants (which he calls 'free variables').

In the course of the paper he makes several points which it is useful to discuss. Bearing in mind his policy of symbolizing all singular terms with individual constants we first turn to what he says about the formula

$$(iv) \quad \Box (\underline{a} = \underline{a}) \supset (E\underline{x}) \Box (\underline{x} = \underline{a})$$

Hintikka maintains that (iv) is false, especially when interpreted in accordance with the notion of possible worlds when

¹J. Hintikka, "Existential and Uniqueness Presuppositions" in Philosophical Problems in Logic, ed. K. Lambert, Riedel Dordrecht, 1970, pp. 20-55.

$\Diamond p$ "is true in a possible world iff p were true in at least one alternative possible world."¹ He says that, in (iv), the antecedent is clearly true, but the consequent can be false. He argues as follows:

Reading ' a ' - 'The next president of the United States', the antecedent of $\llbracket (iv) \rrbracket$ says that necessarily the next president is the next president, which is obviously true. The consequent says that there is someone who necessarily is the next president, i.e. whose election is inevitable. On any reasonable interpretation of necessity, this is false.²

Two points must be made. First, Hintikka has very quickly dropped his interpretation of ' $\Diamond p$ ' as 'in at least one possible world p ' and has lapsed into 'necessarily' and "reasonable interpretation(s) of necessity." Secondly, Hintikka diagnoses why (iv) can be false by saying

Whatever goes wrong with $\llbracket (iv) \rrbracket$ is due to the fact that under different courses of events we consider possible ' a ' refers to different individuals.³

But here he is wrong unless we take it that ' a ' must stand for a definite description.

In our system $M^3Q = (iv)$ is not self-sustaining, but that is simply because although identities are true in all possible worlds it does not follow that some one individual is in all possible worlds. In $M^3Q =$ we read (iv) as

If $a = a$ is true in all worlds then there is some thing from u which is a in all worlds.

The reference of ' a ' does not change, ' a ' does not exist in all worlds. That is why the consequent can be false in $M^3Q =$.

¹Ibid. p. 27.

²Ibid. p. 25.

³Ibid. p. 29.

But, in system $M^1Q =$, where the set of individuals is the same in all worlds, then (iv) is self-sustaining. And (iv) is self-sustaining just because not only is 'a' taken to be a rigid designator, but a is in every world. We can, in $M^1Q =$, only take (iv) to be false if we take 'a' to be a definite description which does pick out a different individual from world to world. Then there is not some one individual from one world which is the x (i.e. a) in all worlds.

At any rate, after discussing (iv) he concludes that the basic semantical idea of 'possible worlds' shows that almost any ordinary language statement in which a singular term occurs within a modal context is in principle potentially ambiguous. Such a statement can sometimes be understood in (at least) two different ways. It can be taken to be about the different individuals which the term picks out in the different possible worlds that the modal operator invites us to consider. However, often it can also be understood as being about the unique individual to which the term in fact refers (i.e. refers in the actual world).¹

He further argues that we should try to prevent singular terms from exhibiting "this kind of referential multiplicity",² and he sets about constructing a logic which will ensure that not only do proper names have uniqueness of reference, but so also do descriptions.

Hintikka's conclusion can only be granted when the singular terms referred to are definite descriptions. In that case, what he says about ambiguity is certainly true. But, if it is the case that ambiguity arises in the case of singular terms which are definite descriptions then it would seem best to try to resolve the problem by a consideration of the logic of definite descriptions in modal contexts. As it is Hintikka attempts to solve the problem by

¹Ibid. p. 28.

²Ibid. p. 29

constructing an elaborate modal logic without any consideration of the theory of descriptions.

This failure to separate proper names from definite descriptions can also be seen in Hintikka's application of (iv) to Quine's example:¹

$$(v) \quad \Box (\text{the number of planets} = \text{the number of planets}) \supset \\ (\exists x) \Box (\text{the number of planets} = x)$$

The singular terms in (v) are definite descriptions, and it is no wonder that one hesitates to say that (v) is logically true. But if we don't treat definite descriptions as rigid designators, and there seems no reason to agree with Hintikka that all singular terms should have unique reference, then we can sustain (iv) but deny (v) the status of logical truth. The basis of our systems has been that there is an invariant relation between names and the individuals named across all possible worlds. There seems to be every reason to deny such an invariant relation across all possible worlds between "the number of planets" and a particular natural number. Our intuitions about (v), when ' \Box ' is read as 'in all possible worlds', is that the number of planets could change from world to world.

Definite Descriptions

We now consider the questions which arise when formulae contain both definite descriptions and modal operators. Expressions such as "the number of planets", "the morning star", "the evening star" and "the time-traveller" are examples of such definite descriptions. Such expressions are meant to pick out one individual.

Definite descriptions can be introduced into a formal

¹Ibid. p. 26.

system either by means of a primitive operator or by means of definitions. In what follows we shall follow the outline of the discussion to be found in Hughes and Cresswell,¹ but we should be able to give firmer answers to the questions they raise than the answers they give.

We do not adopt the usual symbolism for definite descriptions because it becomes quite cumbersome as the theory is developed. Rather, we add to the improper symbols the following:

$$\underline{I} \quad \underline{\lceil} \quad \underline{\rceil} \quad ,$$

\underline{I} is a symbol like U and E and is used to build a quantification operator : $(\underline{I}x)$. This operator is a binary quantification with a scope of the form $\underline{\lceil} \phi(\underline{x}), \psi(\underline{x}) \rceil$ where $\phi(x)$ is the result of substituting \underline{x} for at least one occurrence of an individual constant in the formula ϕ . There is a sense in which

$$(\underline{I}x) \underline{\lceil} \phi(\underline{x}), \dots (\underline{x}) \rceil$$

can be read as

'the ϕ '.

The formula

$$(\underline{I}x) \underline{\lceil} \phi(\underline{x}), \psi(\underline{x}) \rceil$$

is read as

'the ϕ is ψ '.

and the formula

$$(\underline{I}x) \underline{\lceil} Mx, x = a \rceil$$

is read as

'the morning star is Venus',

provided 'M' stands for the property of being a morning star and ' \underline{a} ' stands for the name 'Venus'.

¹Op.cit. pp. 202-209.

If we expand the systems $M^iQ = (1 \leq i \leq 4)$ by the addition of the symbols listed above we will need to add suitable formation rules, and suitable consistency rules for model systems. First we construct formation rules:

(we use $\underline{A}(\underline{x}^1/\underline{a})$ for the result of substituting \underline{x} for at least one occurrence of \underline{a} in \underline{A} .)

- (6.I') If \underline{A} is a wff and \underline{B} is a wff and \underline{x} is any individual variable and \underline{a} and \underline{b} are any individual constants, not necessarily distinct, then

$$(\underline{I}\underline{x})/\underline{A}(\underline{x}^1/\underline{a}), (\underline{B}(\underline{x}^1/\underline{b}))\overline{]} \text{ is a wff,}$$

provided that all the individual constants in $\underline{A}(\underline{x}^1/\underline{a})$ and $\underline{B}(\underline{x}^1/\underline{b})$ have the same superscript unless they occur in atomic parts of the form of an identity.

- (6.IM) If \underline{A} is a wff containing no modal operators and \underline{B} is a wff containing no modal operators and \underline{x} is any individual variable and \underline{a} and \underline{b} are any individual constants not necessarily distinct, then

$$(\underline{I}\underline{x})/\underline{A}(\underline{x}^1/\underline{a}), (\underline{B}(\underline{x}^1/\underline{b}))\overline{]} \text{ is a wff,}$$

provided that all the individual constants in $\underline{A}(\underline{x}^1/\underline{a})$ and $\underline{B}(\underline{x}^1/\underline{b})$ have the same superscript.

- (6.I) is the same as (6I') but without the proviso.

We expand the formation rules of $M^1Q =$ by adding (6I), and those of $M^2Q =$ and $M^3Q =$ by adding (6I'), and those of $M^4Q =$ by adding (6.IM).

We then have formulae for the systems M^iQI ($1 \leq i \leq 4$). We shall use $\underline{A}(\underline{x})$ and $\underline{B}(\underline{x})$ in just the same sense of $\phi(\underline{x})$, and $\underline{A}(\underline{a}^i)$ for a formula, \underline{A} , containing at least one occurrence of an individual constant with the superscript i : \underline{a}^i .

In any formula of the form $(\underline{I}\underline{x})/\underline{A}, \underline{B}\overline{]}$ the first four symbols, $(\underline{I}\underline{x})$, are called 'the description operator', and $\underline{A}, \underline{B}\overline{]}$

is the scope of $(\underline{I}x)$.

The conditions for membership of model sets are drawn from the following:

$$(C.I \sim) \quad (\underline{I}x) \underline{\overline{A}}, \sim \underline{\overline{A}} \notin \mu_n$$

$$(C.I_n^-) \quad \text{If } (\underline{I}x) \underline{\overline{A}}, \underline{\overline{B}} \in \mu_n \text{ and } (\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n \text{ then } (\underline{B}(\underline{a}^n/\underline{x})) \in \mu_n.$$

$$(C.I^-) \quad \text{If } (\underline{I}x) \underline{\overline{A}}, \underline{\overline{B}} \in \mu_n \text{ and } (\underline{A}(\underline{a}/\underline{x})) \in \mu_n \text{ then } (\underline{B}(\underline{a}/\underline{x})) \in \mu_n.$$

$$(C.II) \quad \text{If } (\underline{I}x) \underline{\overline{A}}, \underline{\overline{B}} \in \mu_n \text{ and } (\underline{A}(\underline{a}/\underline{x})) \in \mu_n \text{ and } (\underline{A}(\underline{b}/\underline{x})) \in \mu_n \\ \text{then } \underline{a} = \underline{b} \in \mu_n$$

$$(C.I \text{ self } +) \quad \text{Not } (\underline{I}y) \underline{\overline{A}}(\underline{y}), (\underline{I}x) \underline{\overline{A}}(\underline{x}), \underline{x} \neq \underline{y} \in \mu_n, \text{ provided} \\ \text{that } \underline{A}(\underline{x}) \text{ is the same as } \underline{A}(\underline{y}) \text{ except for having} \\ \underline{x} \text{ wherever } \underline{A}(\underline{y}) \text{ has } \underline{y} \text{ occurring.}$$

$$(C.I =) \quad \text{If } (\underline{I}y) \underline{\overline{A}}(\underline{y}), (\underline{I}x) \underline{\overline{B}}(\underline{x}), \underline{x} = \underline{y} \in \mu_n \text{ and} \\ (\underline{I}x) \underline{\overline{A}}(\underline{x}), \underline{\overline{C}}(\underline{x}) \in \mu_n \text{ and } \underline{A}(\underline{x}) \text{ is the same as} \\ \underline{A}(\underline{y}) \text{ except for having } \underline{x} \text{ wherever } \underline{A}(\underline{y}) \text{ has } \underline{y} \text{ occurring,} \\ \text{then}$$

$$(\underline{I}x) \underline{\overline{B}}(\underline{x}), \underline{\overline{C}}(\underline{x}) \in \mu_n.$$

$$(C.I_n =) \quad \text{If } (\underline{I}y) \underline{\overline{A}}(\underline{y}), (\underline{I}x) \underline{\overline{B}}(\underline{x}), \underline{x} = \underline{y} \in \mu_n \text{ and} \\ (\underline{I}x) \underline{\overline{A}}(\underline{x}), \underline{\overline{C}}(\underline{x}) \in \mu_n \text{ and } \underline{A}(\underline{x}) \text{ is the same as} \\ \underline{A}(\underline{y}) \text{ except for having } \underline{x} \text{ wherever } \underline{A}(\underline{y}) \text{ has } \underline{y} \text{ occurring,} \\ \text{then}$$

$$(\underline{I}x) \underline{\overline{B}}(\underline{x}), \underline{\overline{C}}(\underline{x}) \in \mu_n \text{ provided that all the} \\ \text{individual constants in } \underline{A}, \underline{B} \text{ and } \underline{C} \text{ have the same} \\ \text{superscripts } i \text{ where } i \geq 1.$$

$$(C.I_n) \quad \text{If } (\underline{I}x) \underline{\overline{A}}, \underline{\overline{B}} \in \mu_n \text{ then } (\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n \text{ for at least} \\ \text{one individual constant } \underline{a}^n.$$

$$(C.I) \quad \text{If } (\underline{I}x) \underline{\overline{A}}, \underline{\overline{B}} \in \mu_n \text{ then } (\underline{A}(\underline{a}/\underline{x})) \in \mu_n \text{ for at least} \\ \text{one individual constant } \underline{a}.$$

$$(C. \sim I) \quad \text{If } (\underline{I}x) \underline{\overline{A}}(\underline{x}), \underline{\overline{B}} \in \mu_n \text{ then}$$

$$(\underline{U}n)(\underline{A}(\underline{x}) \& \underline{B} \supset (\underline{E}y)((\underline{A}(\underline{x})(\underline{y}/\underline{x})) = \underline{x} \neq \underline{y})) \in \mu_n.$$

We also add the usual definition of $(E! \underline{x}) \underline{A}$:¹

$$(E! \underline{x}) \underline{A} = \text{df } (E\underline{y})(U\underline{x})(\underline{A} = \underline{x} = \underline{y}) \text{ where } \underline{y} \text{ does not occur in } \underline{A}.$$

We then have the rule:

$$(C.IE) \quad \text{If } (E! \underline{x}) \underline{A} \in \mu_n \text{ then } (I\underline{x}) \underline{A}, \underline{A} \in \mu_n.$$

We denote the sets $C^{M^i QI}$ by IC^i ($1 \leq i \leq 4$).

The sets:

$$I = \{ (C.I \sim), (C.I \neg), (C.II), (C.I \text{ self } \neq), (C.IE) \}$$

$$I_n = \{ (C.I \sim), (C.I_n), (C.II), (C.I \text{ self } \neq), (C.IE) \}$$

$$IC^1 = C_M^1 \cup I \cup \{ (C.I =), (C. \sim I) \}$$

$$IC^2 = C_M^2 \cup I_n \cup \{ (C.I \equiv), (C. \sim I) \}$$

$$IC^3 = C_M^3 \cup I_n \cup \{ (C.I \equiv), (C. \sim I) \}$$

$$IC^4 = C_M^4 \cup I_n \cup \{ (C.I \equiv), (C. \sim I) \}$$

If we add to IC^1 the rule (C.I), or add to IC^2 , IC^3 or IC^4 the rule (C.I_n) then the expanded description systems will be designated as $M^i QI^1$ ($1 \leq i \leq 4$) respectively.

From the formation rules we can see that in $M^4 QI$ there will be no modal operators in the scope of any description operator, and all the individual constants in the scope of a description operator will have the same superscript. These features of $M^4 QI$ render the system uninteresting from our point of view.

Once again our interest is centred on $M^1 QI$ and $M^3 QI$. When we have

$$(vi) \quad (I\underline{x}) \underline{A}(\underline{x}), \underline{B}(\underline{x}) \in \mu_n$$

without modal operators or transworld identities in the scope of $(I\underline{x})$, then this simply indicates that there is, in the world described by μ_n , just one individual with the property set out in $\underline{A}(\underline{x})$ and it

¹Cf. Hughes and Cresswell, op.cit.

also has the property set out in $B(x)$. But there is no guarantee at all that the individual which is the A in the world described by μ_n is also the A in any other world, or that in any other world there is any A, or that if there is any A there are not a thousand A's. Even when we have

$$(vii) \quad \Box (Ix) \overline{A(x)}, B(x) \overline{] \in \mu_n},$$

with the same provisos concerning the scope of (Ix) as for (vi), all that (vii) can guarantee is that in all possible worlds the A is B, but not that the same individual in each world is the A which is B. For example, in both the real world and the fictional world of Hannibal's victory the winner of the battle entered Rome in triumph, but the winner in one world was Scipio, in the other it was Hannibal.

Hughes and Cresswell find that when they introduce definite descriptions there are certain difficulties with the logic they construct. They point out that there are certain options open to one when constructing such a logic. Either we can see $(Ix) \overline{A(x)}, \dots (x) \overline{]$ as designating one object in one world (of which something will be predicated), or we can see $(Ix) \overline{A(x)}, \dots (x) \overline{]$ as designating one object which is A in all worlds. In their logic, the former option makes

$$(viii) \quad (E! x) \underline{A} \supset (Ix) \overline{A}, \underline{A} \overline{] \text{ and}}$$

$$(ix) \quad \Box (E! x) \underline{A} \supset (Ex) \Box \underline{A} \text{ to be valid.}$$

The latter option renders (viii) and (ix) invalid, but

$$(x) \quad (E! x) \Box \underline{A} \supset (Ix) \overline{A}, \underline{A} \overline{] \text{ valid. They favour the latter option.}}$$

Taking each of (viii), (ix) and (x) in turn, and assuming them to be members of some μ_n , we would read them respectively as:

$$(viii') \quad \text{If exactly one thing in world } n \text{ is } \underline{A} \text{ then the } \underline{A} \text{ is } \underline{A} \text{ in that world.}$$

- (ix') If in every world exactly one thing is A then at least one thing from world n is, in every world, A.
- (x') If exactly one thing from world n is A in every world then in world n the A is A.

On the basis of these readings we would want (viii) to be valid but not (ix) and (x). In our systems (viii) is self-sustaining simply by virtue of rule (C.IE). A counter-example to (ix) is where, in three worlds n, n' and n'', we have three individuals a, b and c such that:

$$\begin{aligned}\mu_n &= \{ \underline{Aa}, \sim \underline{Ab}, \sim \underline{Ac} \} \\ \mu_{n'} &= \{ \sim \underline{Aa}, \underline{Ab}, \sim \underline{Ac} \} \\ \mu_{n''} &= \{ \sim \underline{Aa}, \sim \underline{Ab}, \underline{Ac} \}\end{aligned}$$

In every world exactly one individual is A, but there is no one individual which is A in all worlds. A counterexample to (x) is

$$\begin{aligned}\mu_n &= \{ \underline{Aa}, \underline{Ab}, \sim \underline{Ac} \} \\ \mu_{n'} &= \{ \underline{Aa}, \sim \underline{Ab}, \sim \underline{Ac} \} \\ \mu_{n''} &= \{ \underline{Aa}, \sim \underline{Ab}, \underline{Ac} \}\end{aligned}$$

Here there is just one individual from n which is A in all worlds, that individual is a. But there is no one and only A (the A) in n. So the antecedent of (x) is true but the consequent is not. Not only is (x) not self-sustaining, but neither is

$$(xi) \quad (E!x) \Box A \supset (E!x) A$$

The same counterexample falsifies (xi) as falsified (x).

Quine's problematic (v)¹ in the notation of definite descriptions becomes:

$$\begin{aligned}(xii) \quad & (\underline{Iy}) \neg N_y, (\underline{Ix}) \neg N_x, x = y // \\ & (Ex) \quad (\underline{Iy}) \neg N_y, x = y\end{aligned}$$

where 'N_x' stands for something like 'x is a number arrived at by

¹See page 87 above.

counting the planets.¹ Since the antecedent of (xii) is self-sustaining by virtue of rule (C.I self \neq) and the modal rules, we need only look to see what the status of the consequent is. A counterexample to this can be constructed by setting out a series of worlds in which there are differing numbers of planets.

If the predicate, N , does not concern numbers, for example, if ' Nx ' were to stand for ' x is a letter written by Socrates', then the consequent of (xii) could be rendered false simply by there being no such letter in any μ_n . This sort of counterexample is not available in $M^i QI$ ($1 \leq i \leq 4$) because of rules (C.I) and $C.I_n$. Nevertheless (xii) is not self-sustaining in those systems either. Counterexamples of the former kind are available, that is where different individuals from world to world have the property of being a letter written by Socrates.

The one major difference between the treatment here and Hughes and Cresswell's treatment of the description operator as primitive is that we have not extended the Universal instantiation rules to include:

(C.UI) If $(\underline{Ux}) \underline{A}(\underline{x}) \in \mu_n$ then $(\underline{Ix}) [\phi(\underline{x}), \underline{A}(\underline{x})] \in \mu_n$

The extension of the axiom V1 in Hughes and Cresswell to

$((\underline{Ux}) \underline{A} \supset \underline{B})$ whenever \underline{B} differs from \underline{A} only in having any term in place of a free occurrence of \underline{x} in \underline{A} (provided that no variable free in the term is bound in \underline{B})²

is, in our system, the same as adopting (C.UI). There is a difficulty in adopting (C.UI). Even if ' ϕ ' is a monadic predicate, the rule must be seen as applying to all monadic predicates. So, unless

¹A precise formalisation would require the use of set theory e.g. ' Nx ' stands for ' $x = C\{y | y \text{ is a planet}\}'$

²Op.cit. p. 203.

$(\underline{Ux}) \underline{A}(\underline{x})$ is never satisfiable, (C.UI) means that in every world there is just one ϕ . One way of avoiding this unpleasant conclusion is to drop rule (C.I~) in our systems, a move which is analogous to Hughes and Cresswell's stipulation that the value of 'the ϕ ' should be the one individual which is the ϕ , or else some arbitrary individual.¹ This can only be seen as highly counter-intuitive, just as allowing "The ϕ is not ϕ " to be satisfiable can only be seen as highly counterintuitive. We can only say that it seems preferable to have rule (C.I~), and so it seems preferable to use systems of the style of $M^i QI$ rather than $M^i QI^1$ ($1 \leq i \leq 4$).

Definite descriptions can also be introduced by definition. In Hughes and Cresswell² the treatment is confined to monadic predicates. The definitions would be

$$(A) \quad (\underline{Ix}) [\phi \underline{x}, \psi \underline{x}] = \text{df } (E' \underline{x}) \phi \underline{x} \ \& \ (\underline{Uy}) (\phi \underline{y} \supset \psi \underline{y})$$

and

$$(B) \quad (\underline{Ix}) [\phi \underline{x}, \underline{x} = \underline{a}] = \text{df } \phi \underline{a} \ \& \ (\underline{Ux}) (\phi \underline{x} \supset (\underline{x} = \underline{a})).$$

Two things emerge from Hughes and Cresswell's discussion. The first has to do with the formulae which are logical truths when definite descriptions are introduced by definition rather than by taking the description operator as primitive. The second has to do with the ambiguities of the notation which Hughes and Cresswell use. They use the traditional Russellian notation where 'the ϕ is ψ ' is formalised as ' $\psi (\iota \underline{x}) \phi \underline{x}$ '. Our notation simply avoids all such ambiguity. There are other ways of avoiding the ambiguity also,³

¹Op.cit. p. 203 ff.

²Op.cit. p. 207.

³See A. F. Smellyan, "Modality and Description", The Journal of Symbolic Logic, Vol. 13, 1948, pp. 31-37.

chiefly by means of scope indicators. Because our notation avoids the ambiguity we will concentrate on the differences which result from the differing approach.

In the systems $M^i QI^1$ ($1 \leq i \leq 4$) both (A) and (B) are self-sustaining. And just as, in the definitional system set out in Hughes and Cresswell,

$$(xiii) \quad (Ix) \neg \phi x, x = a \supset \Box (Ix) \neg \phi x, x = a$$

is not valid, it is not self-sustaining in $M^i QI^1$ or $M^i QI$ ($1 \leq i \leq 4$).

But more importantly, (xii) is not self-sustaining in this definitional version of the theory of descriptions.

It can be seen from this explicit treatment of definite descriptions as definite descriptions that we can deal with the opposite problems found by Quine in the form of (v).

Empty definite descriptions

Finally, before leaving the subject of definite descriptions we need to consider what is involved, in more detail, in refraining from including (C.U1) in our preferred systems which deal with definite descriptions (not by definition).

We have allowed ' $(Ix) \neg \phi (x), \dots (x) \neg$ ' to be non-designating as well as designating, and both those possibilities are open within the one possible world, that is when no modal operators occur in ' $(Ix) \neg \phi (x), \psi (x) \neg$ '. When we look at $M^1 QI$ in the light of these features of formulae containing definite descriptions we see many similarities between $M^1 QI$ and Thomason's system Q3.¹ The major difference between $M^1 QI$ and Q3 is that Q3 is a free logic. Non-designating

¹R. H. Thomason, "Modal Logic and Metaphysics" in The Logical Way of Doing Things, Yale University Press, London, 1969, pp. 119-146, esp. pp. 138-145.

definite descriptions are given as a value some object is a domain of non-existent objects. Non-designating free-variables (constants) are similarly treated (and allowed for).

Thomason says that he wishes to "take the sort of reference in which a name ('Socrates') is assigned one thing (Socrates) which is the same in many possible worlds as primary or paradigmatic."¹ This is, of course, one of the presuppositions according to which we are constructing our systems. These individuals-in-every-possible-world Thomason claims are substances.

Where he differs from us is that in making the logic free he only allows the existing objects to be common to every possible world, but allows for each world a domain (set) of individuals to be the values of non-designating free variables (constants) and definite descriptions.

This makes Q3 very much like ²MQI where the set of individuals common to all worlds could be called substances. If we make our logics into free logics then we add additional flexibility to our systems, and we consider the question of free logics in a later chapter. But we return to the question of substance later in this chapter.

At this point we should simply note that definite descriptions, like proper names, purport to refer. So it seems only reasonable that if we assert that (in world n) the ϕ is such and such then we are asserting something about the ϕ . Up to this point we have been at pains to show that definite descriptions are different to names. Indeed, it might be said that a definite description carries its sense explicitly while a name does not. A definite description should not only be understood to be a singular

¹Ibid. p. 137.

term, but also, that to which it refers should be explicitly described. With a name we might only understand that it does refer uniquely but not have set out explicitly by description anywhere that to which it refers.

It is the descriptiveness of definite descriptions which particularly distinguishes them from names. But both names and definite descriptions are assumed to have, in any one world, uniqueness of reference.

Individuating Functions & Rigid Designators

Hintikka's case for the need for the highly complex modal logic which he develops in "Existential and Uniqueness Presuppositions" is most certainly based on the confused conflation of proper names and definite descriptions. None the less, there is another important theme which runs through Hintikka's discussion of singular terms and cross-world identification. That theme is that whatever individual constants are symbolizing in a notation the constants themselves should be seen as having the same referent from world to world.¹

In another paper, "Semantics for propositional attitudes",² Hintikka gives his answer to the question he poses, "With what right do we speak of individuals in different possible worlds as being identical?" But before giving his answer he insists that he is discussing possible worlds in as much as they assist in illuminating propositional attitudes such as belief and knowledge. His interest is primarily in epistemic modalities such as are expressed by "a

¹Ibid. p. 29.

²J. Hintikka, "Semantics for Propositional Attitudes" in Philosophical Logic, Eds. J. W. Davis, D. J. Hockney, W. R. Wilson, Reidel Dordrecht, 1969, pp. 21-45.

believes that p " (and symbolized by ' $B_a p$ '). At the same time, the suggestions he makes about cross world identification are eventually applied to alethic modalities in "Existential and Uniqueness Presuppositions".

Because of Hintikka's primary interest in epistemic modality we will first give an example of how the problem of cross world identity arises in epistemic contexts. For example, suppose that a believes that Adam was the first male homo sapiens and also that Adam was six feet tall, brown skinned, and lived for several hundred years. Now, it would be compatible with a 's beliefs about Adam that Adam was blue-eyed and lived for 756 years, but it would also be compatible with a 's beliefs that Adam was brown-eyed and lived for 656 years. So Hintikka suggests that besides the world in which a is supposed actually to live and have beliefs, we also suppose that there are "alternate" possible worlds in which two things obtain. First, what a believes is actually true, and second, what is compatible with a 's beliefs (or what we maintain is compatible with a 's beliefs) is also true.

So we have three model-sets, to fit our example, which can be μ_n , μ_m , and μ_j . In μ_n are the statements:

a believes that Adam was the first male homo sapiens, was six feet tall, brown skinned, and lived for several hundred years.

It is compatible with what a believes that Adam was blue-eyed and lived 756 years.

It is compatible with what a believes that Adam was brown-eyed and lived 656 years.

In μ_m , an alternative to μ_n , are the statements:

Adam was the first male homo sapiens, was six feet tall,
brown skinned, and lived for several hundred years.

Adam was blue-eyed and lived for 756 years.

In μ_j , an alternative to μ_n , are the statements:

Adam was the first male homo sapiens, was six feet tall,
brown-skinned, and lived for several hundred years.

Adam was brown-eyed and lived for 656 years.

The important question that arises is, by what right do we assume that the blue-eyed 756 year old man in one world is the same person as the brown-eyed 656 year old man in another world? To press the question further, we might ask, is it not possible that we have two separate individuals given the same name?

Hintikka says that in the case of propositional attitudes possible worlds are

...normally possible states of affairs or courses of events compatible with the attitude in some specified person. Now normally these attitudes may be attitudes towards definite persons as definite physical objects. But how is it that we may be sure, sight unseen, that the attitudes are directed toward the right persons or objects? Only if in all the possible worlds compatible with the attitude in question we can pick out the recipient of this attitude, i.e. the individual at its receiving end.¹

So, if a 's belief is towards (or about) Adam then we must ensure that Adam in the world partially described by μ_n is the same individual as the Adam in the worlds described by μ_m and μ_j . Hintikka then postulates "a set of functions \underline{f} each member \underline{f} of which picks out at most one individual $\underline{f}(\mu)$ from the domain of individuals Furthermore, we must often require that, given $\underline{f}_1, \underline{f}_2 \in \underline{F}$, if $\underline{f}_1(\mu) = \underline{f}_2(\mu)$ then $\underline{f}_1(\lambda) = \underline{f}_2(\lambda)$ for all alternatives λ to μ . In other words, an individual cannot 'split'

¹Ibid. p. 34.

when we move from a world to its alternatives."¹ The functions in \underline{F} are called "individuating functions."

Hintikka then claims that the question of when an individual in one world is the same individual as one in another is not as important as whether there is a function in \underline{F} which picks out both individuals. So $\underline{a} = \underline{b}$, when \underline{a} is in world \underline{n} and \underline{b} in world \underline{m} , iff there is a function in \underline{F} such that that function, say \underline{f} , gives $\underline{f}(\underline{n}) = \underline{a}$ and $\underline{f}(\underline{m}) = \underline{b}$. There is a sense in which, Hintikka says, these functions can be seen as the names of the individuals which appear (or exist) in a variety of possible worlds — "the members of \underline{F} might in fact be thought of as names or individual constants of a certain kind."²

Having proposed all this, Hintikka then qualifies it all by pointing out that in some modalities we might want individuals to split and merge. Furthermore he points out that the members of \underline{F} are not themselves individuals manifesting themselves in various possible worlds. The members of \underline{F} can be seen as embodying the totality of ways of recognising "one and the same individual under different circumstances and under different courses of events."³

In as much as the members of \underline{F} determine whether two individuals in different worlds are one and the same or not, there is some point in seeing the members of \underline{F} as names or individual constants. Then, by definition, or at least by presupposition, these names are rigid designators. What a member of \underline{F} picks out from world to world is taken as being the one individual, the referent, as it were, of

¹Ibid. p. 34.

²Ibid. p. 35.

³Ibid. p. 35.

the name.

This use of members of \underline{F} as rigid designators is at one with Rennie's account of rigid designators.¹ In Hintikka's account of \underline{F} it is set down that \underline{F} is a set of functions from worlds to individuals. In Rennie, where D_1 is the set of all individuals and D_k is the set of all worlds, the set D_{ik} is the set of all functions from worlds to individuals. Of D_{ik} , Rennie says

we might expect D_{ik} to form a domain of intensional individuals, or individual concepts. That is, an individual concept is an indexed sequence of individuals ... this usage of the term is very similar to that of Carnap.²

Hintikka also refers to Carnap and says of the individuating functions,

They are excellent approximations in our theory to the 'individual concepts' which many philosophers have postulated.³

There is then a footnote reference to Carnap⁴ which is the same as Rennie's footnote to the section quoted above. Rennie defines a rigid designator, in the sense of Kripke, as a function from worlds to individuals.⁵ He then says that "a rigid designator is a designator ... that takes on the same substance in each world."⁶

In spite of the great similarity between Rennie's and Hintikka's accounts of rigid designators there is one vital underlying

¹Op.cit., p. 91 ff.

²Ibid. p. 91.

³Op.cit. p. 39.

⁴R. Carnap, Meaning and Necessity, University of Chicago Press, Chicago, 1947.

⁵Op.cit. p. 92.

⁶Ibid. p. 93.

difference. In Rennie's account of rigid designation the definition of a rigid designator makes use, in the definiendum, of ordinary identity. What Rennie's definition says, substantially, is that a rigid designator is a function such that there is an individual which in all worlds is the individual picked out by the function. So if a function does not pick out the same individual in all worlds it is not a rigid designator. The notion of 'same individual' is primitive. By contrast, in Hintikka's account of individuating functions it is the function which determines identity. The individuating function is primitive. This very contrast is discussed by Rennie, where Rennie says of Hintikka that

he says that merely to assume that the same individuals can crop up in different worlds is just to wish the problem [of cross-world identification] away, not to solve it. His solution to the problem is to postulate the existence of what we might call a canonical set of individual concepts.¹

Despite this one important difference between Hintikka's account and Rennie's account of rigid designators, there are similarities such that there is one important problem for both accounts. The problem is that both assert that names are functions. This just does not seem appropriate.

We have already argued that names are items in a syntactic category, and are symbols. This is not to say that names are typographic entities, just as numbers are not typographic entities. Names, as symbols, stand in a relation to what is named, and although we have argued that the ideal naming relation is a function, names themselves are not functions. In a way, our argument is that the members of F are the senses of names.

Hintikka is aware of this possibility and says

¹Ibid. pp. 112 ff.

The functions that belong to \underline{F} may of course be considered special cases of the 'individual concepts' postulated by some philosophers of logic or as special cases of Frege's 'senses' (*Sinne*). No identification is possible between the two classes, however, for we saw earlier that not every arbitrary singular term (say b) which picks out an individual from each $\underline{\text{set}}$ of individuals in a world/ we are considering goes together with an $\underline{f} \in \underline{F}$, although every such term is certainly meaningful and hence has a Fregean 'sense' and perhaps even gives us an 'individual concept'. As I have put it elsewhere, members of \underline{F} do not only involve a 'way of being given' as Frege's senses do, but also a way of being individuated.¹

Here, it is clear that Hintikka has either dropped the idea that names are members of \underline{F} , or his original suggestion placed great emphasis on the qualification that such functions were names of a certain kind.

It seems to me that it would not take much change in Hintikka's point of view for that point of view to become the view that the members of \underline{F} are the senses of names. This is not to say that any changes would be unimportant, they would be important, but the view is close to the view being advocated herein. We have already argued that Hintikka should differentiate between names and definite descriptions. We have also argued that the sense of a name involves both a way of being given and the individuation of that to which the name is (was) given. The most formidable problem, though, is whether we take the identity of individuals to be primitive or the identity of individuating functions to be primitive. If the latter course is followed it should be pointed out that it is usual to show functions to be identical by resort to the identity of individuals, rather than vice versa.

As far as Rennie's account of a rigid designator is concerned there is no clear qualification of what he says about a name's being a function. Indeed, he suggests that the way to formalize

¹Op.cit. p. 39.

a name like 'Nixon' is to use a functional expression: $\lambda w_k n_i$ — that is the function which picks out the one individual, Nixon, in all worlds.¹ There seems to be some confusion here between use and mention.

Presumably ' $\lambda w_k n_i$ ' designates a function, and ' n_i ' designates the individual who, in each world w_k , is called 'Nixon'. ' $\lambda w_k n_i$ ' is not itself a function, but is the formalization of or the symbolic equivalent of 'Nixon'. If ' $\lambda w_k n_i$ ' designates a function of constant value, then if ' $\lambda w_k n_i$ ' is the symbolic equivalent of 'Nixon' we would expect 'Nixon' to designate a function of constant value (that value being Nixon). But 'Nixon' designates Nixon, not the function whose constant value is Nixon.

It might be suggested that ' $\lambda w_k n_i$ ' designates a relationship, de re, between worlds. That relationship is the relationship which pertains between a series of individuals in worlds. But if that were the case then ' $\lambda w_k n_i$ ' would seem to be inappropriate, for its set theoretic expansion would be

$$\{ \langle n_i, w_1 \rangle, \langle n_i, w_2 \rangle, \langle n_i, w_3 \rangle, \dots, \langle n_i, w_n \rangle, \dots \} \\ (n \geq 1)$$

which is quite different to the sort of theoretic expansion which one would give to a relationship between individuals in various possible worlds, in particular, for the Nixon case we would get either

$$\{ \langle n_i, n_i \rangle \}$$

or

$$\{ \langle n_i, w_1, n_i, w_2 \rangle, \langle n_i, w_2, n_i, w_3 \rangle, \dots \}$$

depending on how one wanted to describe the relationship. In the

¹Op.cit. p. 94.

former description of the relation we take it that Nixon is one and the same person (individual) in all worlds, so the relation is simply that of identity. But there is here a presupposition of rigid designation. The name 'Nixon' simply names that person who is one and the same person in all worlds, so that it is true that $(n_i = n_i)$. But that can hardly be Rennie's view, and anyway ' $\lambda w_k n_i$ ' is not identity.

In the latter description we are really adopting a contingent identity account of cross-world identification.¹ Such an account would rely on propositions of the form of Nixon in world 1 is Nixon in world 2. Once again, this is not the relation designated by ' $\lambda w_k n_i$ '.

The confusion in Rennie's account of rigid designation is somewhat compounded by the fact that he has but one domain of individuals, D_i , the members of which he designates by symbols such as ' n_i ' which can not really be taken to be anything other than individual constants. So the symbol ' n_i ' is going to designate one and the same individual -- across all possible worlds. In Appendix I we have tried to avoid the confusion which arises here in Rennie's monograph by introducing a type for symbols, or a quotation operator.

Presuppositions and Formalization

In this chapter we began with certain presuppositions and constructed formal systems to give expression to those presuppositions. Our assumptions were first, that constant singular terms were rigid designators; second, that the domain of quantification for quantifiers outside the scope of modal operators would be world

¹Cf. Hughes & Cresswell, op.cit. pp. 189 ff.

relative; and third, that the modal operators were to be interpreted as, in a sense, quantifiers whose domain was to be possible worlds.

In the light of these assumptions certain things became clear about the model-set model-system formalization we used. Our first assumption indicated that identity had to be given special status, and that trans-world identities and non-identities would have to be able to appear in any (partial) description of any world. Unless identity is given this special status then we cannot be sure that our formal systems give accurate expression to our assumptions about the sets of individuals in worlds (Cases I to IV).

The second assumption forced us to be clear about the domains of quantification. The relationships between the sets of individuals in the worlds (partially) described by model sets in model systems had to be spelt out clearly. Once this was done, the formal systems were constructed to match the various possibilities set out in cases I to IV.

The third assumption indicates a way of interpreting the notion of logical possibility, given the other assumptions. But it is not clear that the modality is of logical necessity, whatever that may be. Using the desired interpretation of modal operators we have been able to interpret quantified modal contexts, both in the cases where the quantification is not in the scope of any modal operator but binds some occurrence of a variable which is in the scope of a modal operator, and in the cases where the quantification is itself in the scope of a modal operator.

The most important feature of all this formalizing has been that the interpretation of individual constants as rigid designators has not been changed when the constants occur in modal contexts. Even so, there are some questions which need further

discussion. These questions have already been raised or mentioned.

The first major question for further discussion is that of essentialism. Do individuals have certain essential properties which, from world to world, ensure the sameness of the individual? This will be discussed in the next chapter. The second question concerns those individuals said not to exist. In some of our modal systems we have allowed for the possibility that an individual might exist in one world but not in another. We have said, explicitly that names might be given to non-existent persons, places and things. To what then do such names refer? This will be discussed in the fifth chapter. Finally, there are a set of closely related questions to do with knowledge, acquaintance, quantification into epistemic contexts, and the difference in cognitive value between someone's knowing that $\underline{a} = \underline{a}$ and knowing that $\underline{a} = \underline{b}$. These questions are discussed in the final chapter.

CHAPTER FOUR

ESSENTIALISM

Common Properties

We introduced modal operators and constructed the logics $M^iQ = (1 \leq i \leq 4)$ in order to explore the possibility that there might be some essential features which identify a given individual and which he (it) has in every world in which he (it) appears. At this point we must therefore discuss what is bound up in the notion of an individual's having essential properties.

Marcus distinguishes between two kinds of essentialism which are to be called individuating and Aristotelian. "For Aristotelian essentialism, an essential property is a property an object must have. It answers to the question "What is it?" in a strong sense; if it ceased to have that property it would cease to exist. It is a property such that, if anything has it at all, it has it necessarily."¹

But also "among the attributes an object must have are not only those which it shares with objects of its kind (Aristotelian essentialism), but those which are partially definitive of the special character of the individual and distinguish it from some objects of the same kind."² These individually definitive properties are individually essential.

Marcus claims that we appeal to the former essentialism

¹R. B. Marcus, "Essential Attribution", Journal of Philosophy 65 (1971), p. 190.

²Ibid. p. 191.

when "we say of Moby Dick that although he lives in the sea he is essentially a mammal," but would presumably appeal to the latter when we say of Moby Dick that to find him it is essential that we find a great white whale. Marcus uses as examples of individuating essentialism "Unlike the rest of use, Winston is essentially a cyclist, not a mathematician," and "He's essentially a philosopher, not a politician."

Essentialism, under this view, is useful for sorting the properties of objects. There are those properties which tell us of what kind an object is, crudely put, whether something is animal, mineral or vegetable. There are those properties which show us just who or what something is.

Now it seems to me that Marcus is correct in claiming that essentialism, in some rather vague sense, is a way of sorting properties. For all the examples given, some one property is pointed to in spite of some other property or properties. The adverb "essentially" might therefore be seen as a part of an expression rather than as a basic expression. Consider: "although x is ϕ essentially x is ψ ."

In the case that we set out above, where we had a man waking up in the Roman Empire of 200 B.C. and reckoning himself to be a time traveller, he would probably say that although he had a different body and lived in a different era he was essentially the same person (or it was not absurd to claim that he was essentially the same person).

In general, a great deal depends on why we want to sort the properties of an individual. We may want to classify sea-creatures,

or to identify one particular whale by some simple means. We may want to show how a wise man could be so careless of public opinion, or we may want to give an explanation of what appears to be a man's memories.

So we single out those properties which we count as important for some reason. We began with modal operators in order to explore the possibility that there might be some essential features which identify a given individual. But, in the light of what has just been pointed out, it might be better to phrase our topic as "the possibility that there might be some features essential for the identification of a given individual." Having rephrased the question in this way we must note that we have, in a sense, narrowed the question down to a more or less epistemological question.

Certain important properties are usually used as a means of identifying an individual in the actual world, so perhaps those properties can be counted as the essential properties by means of which the individual is to be identified in other possible worlds. But this will not do, as Kripke has pointed out. Important and essential properties should not be confused.

Kripke writes,

To me Aristotle's most important properties consist in his philosophical work, and Hitler's in his murderous political role; both ... might have lacked these properties altogether. Surely there was no logical fate hanging over either Aristotle or Hitler which made it in any sense inevitable that they should have possessed the properties we regard as important to them; they could have had careers completely different from their actual ones. Important properties of an object need not be essential, unless 'importance' is used as a synonym for essence; and an object could have had properties very different from its most striking actual properties, or from the properties we use to identify it.¹

¹Op. cit. p. 289.

It is often just those important properties which we wish to conjecture about when we say of Aristotle that he might have done so and so. We can say, with perfectly good sense, that Hitler might never have risen to power.

Although we use important properties as an easy way of identifying the individual who is called 'a', in some other world, a world of conjecture, or fiction, the very property we use to identify the individual could be absent. So we come back to the original question about essential properties. In the first place we were more concerned about those properties which an individual would have in all possible worlds, irrespective of how it was identified in any one world.

So, perhaps it would be better to recast the topic as "the possibility that there might be some properties which an individual has which are essential to its being the same sort of individual and also the same particular individual." For example, we would want to claim of our time-traveller that he is essentially a person, that is the sort of individual he is. Also, he was the particular person who invented a time machine and was born and lived for so many years in certain circumstances.

A partial formalization of our claims about Wells would be set out as

\Box Wells is a person & $\Box \Diamond$ (Wells is an inventor of a time machine, and was born and lived etc.)

The first conjunct could be made into

(i) $(\exists x) \Box (Px \ \& \ x = w)$

provided that 'w' stood for 'Wells'

and 'Px' for 'x is a person.'

It follows in both $M^1Q =$ and $M^3Q =$ that

$$(ii) \quad \Box (Ex)(Px \ \& \ x = w)$$

which may be stronger than was intended. (ii) makes it plain that it follows from (i) that Wells appears in all possible worlds. All we would want to claim is that if Wells appears in any world then he (it) is a person. A weaker claim can be set out as

$$(iii) \quad \Box ((Ex)(x = w) \supset Pw)$$

This will do the job in $M^3Q =$, but not in $M^1Q =$. In the latter system, since every individual in every world is in every other world, it follows that w is in every world. So (iii) should imply (ii), and in $M^1Q =$ that is the case.

It is tempting, in $M^1Q =$, to try to resort to a formula such as

$$(iv) \quad \Box (Ux)(Px \ \& \ \Diamond (Tx \ \& \ Lx) \ . \supset \ . \ x = w)$$

where ' Tx ' stands for 'x invented a time machine' and ' Lx ' stands for 'x was born and lived etc.' But (iv) can be satisfied by Wells' not being a person or not in any world inventing a time machine etc. And if we want being a person to be an essential property of Wells then we cannot accept (iv) in $M^1Q =$.

The discussion of these four formulae does little more than demonstrate that the notation allows us to formalize statements about an individuals' having some property in all possible worlds, and having some property in one possible world. But none of the systems we have constructed give any formal basis for distinguishing between the essential and the non-essential properties of any individual.

We can say what we please. The problem is that it is not clear just what one expects of a logic in this situation. There are two alternatives. We might on the one hand, be asking that the

essential properties of an individual be the properties which, when predicated of that individual will yield logical truths of the form $\Box \phi \underline{a}$. On the other hand we might simply want a notation in which we can say of \underline{a} that \underline{a} has certain properties in all possible worlds.

The former demand might be just too much to ask of any logical system. In speaking about the reasonable idea that Julius Caesar is essentially a man, Wiggins writes,

Of course the inconceivability of Julius Caesar's not being a man is not logical inconceivability. The point of calling a sentence a logical truth is that its denial can be shown by logic alone to involve contradiction. A logical truth is a truth forced upon us by the meanings of the logical constants. By this criterion not even 'all bachelors are unmarried' qualifies. For 'bachelor' is no more a logical constant than 'Caesar' is.¹

In a footnote to this passage Wiggins draws a distinction between what is logically possible and what is conceivable or conceptually possible.² Whatever one may want to say about this distinction in the long run, it highlights the fact that in our logical systems the logical constants do not go beyond a formal account of negation, disjunction, 'some' and 'all', identity, 'in all possible worlds', and what can be formulated in terms of these.

Apart from identity there is no predicate for which consistency conditions have been given in our systems, nor is it usual in any standard first order logic to have any other predicate constants. It is in order to compensate for this that Wiggins develops the essentialist theory that he does in "Essentialism, Continuity, and Identity."³

¹Op. cit. p. 350.

²Ibid. fn. 42, p. 358.

³Ibid. pp. 332-337, 347-349.

The latter demand, for a notation in which we can say of \underline{a} that \underline{a} has certain properties in all possible worlds, is satisfied in our systems. We can certainly use the notation to say of \underline{a} that \underline{a} has some property, ϕ , in all possible worlds. But this is curiously unsatisfactory. Since, in our systems, there is no treatment of particular predicates other than identity, it makes it seem as though an individual's possession of a property in all possible worlds is somehow contingent. The individual just happens to have that property in every world in which it exists. In formal terms, we can express this contingency by saying that we can construct a model system such that ' \underline{a} ' occurs in every model set in the system and in a formula ' $\Box \phi \underline{a}$ ' in every model set in the system but ' $\Box \phi \underline{a}$ ' is not self-sustaining. It is this non-self-sustainability which makes for the unsatisfactory nature of the systems for dealing with essential properties.

Because of these difficulties we now turn to the other side of the coin, so to speak. We now take up the question of an individual's properties being radically different in some possible world.

Radical differences

Let us now consider just what is involved in saying that the individuals can be radically different from world to world. We turn to a consideration of the notions bound up in the expressions "individual" and "one and the same individual". If, for the moment, we could agree that we understand the first part of the sentence "Each individual constant in any one system, Ω , will always

designate one and the same individual" then we would need to set out just what is bound up in the latter portion of the sentence.

In the science-fiction story which we have been using we have been concerned to say that Wells^g is one and the same individual as Wells^f. But later we wanted to know just what this amounted to. In our later remarks we did take it for granted that Wells^g was the same person as Wells^f. Had we been completely neutral in this matter we need not have supposed that the individual (in the technical sense) designated by "Wells^g" had any of the properties predicated of Wells^g in μ_g when designated by "Wells^f" and so described in μ_f . But this is just the point at which one balks.

It just does not at first seem to be conceivable, for example, that some individual could be a person, with certain memories, with self-consciousness, and with a human body in one possible world, but be, for example, a ray of light travelling on in a vacuum for ever in some other possible world. On first sight there seems to be a range of properties which can be varied when making the claim that w^g is one and the same individual as w^f , but there seems to be a range of properties which cannot be so easily varied. Partly, it seems as if, for any individual, it would cease to exist if it no longer had some particular property, as Marcus contends.

Kripke contends the same, to all intents and purposes, in the course of discussing a rather elaborate case of the Queen of England supposedly not being the daughter of King George VI and Elizabeth.¹ The case he discusses is, briefly, that a hitherto unheard of daughter of Mr. and Mrs. Truman has been passed off as the daughter

¹Op. cit., pp. 312-314.

of George VI and Elizabeth. The Trumans' daughter is now the Queen of England. Then Kripke asks.

Now, one question is, in this world, was Elizabeth herself ever born? Let's suppose she wasn't ever born. It would then be a situation in which, though Truman and his wife have a child with many of the properties of Elizabeth, Elizabeth herself didn't exist at all.

Kripke then adds,

One can only become convinced of this by reflection on how you would describe this situation.¹

One can only suppose that by this last sentence Kripke means to indicate that were the situation described discovered to be true the usual way of making the situation known would be by saying something like, "George VI and Elizabeth have never had any daughter called 'Elizabeth'. The present Queen of England is really Elizabeth Truman, not Elizabeth Windsor. There is no such person as Elizabeth Windsor." Even though this is a reasonable supposition, it seems just as reasonable to assume that the disclosure of this amazing fact would run like, "Elizabeth Windsor is the adopted daughter of George VI and Elizabeth. The Queen is not of royal blood, etc., etc."

In this case 'Elizabeth Windsor' has been firmly established as the name of the individual who, in the actual world, is the occupant of the throne of England. The sense of the name is firmly established, and would be just as firmly established in the possible world described. Kripke leans very much toward the idea that, for people, the name given at birth is the rigid designator of that person. It may well be the case that, for people, the name given at birth is a rigid designator, but other names may also be

¹Ibid. p. 313.

given, at adoption for example. So it is unlikely that Elizabeth Windsor would be said not to exist, but rather than Elizabeth Windsor was the daughter of Mr. and Mrs. Truman, or even, that Elizabeth Windsor was Elizabeth Truman.

Wiggins also takes Kripke up on this point "that anything coming from a different origin would not be this object."¹ What this amounts to, says Wiggins, is that Kripke wants a prohibition on 'backwards' conditionals. We are to be prohibited from speculating, for example, about a person in such a way as to alter their origins. We cannot really construct a possible world in which the Julius Caesar of this world is born of different parents. "But", asks Wiggins rhetorically, "is there really any such principle limiting the construction of possible worlds?"² It is not clear that there is. Kripke's claim that by speculation about origins other than those possessed by an individual we would "'lose' the very object we mean to be speculating about"³ has not been clearly demonstrated.

Yet, even if we set aside Kripke's claim about origins there is his claim that he wants also a constraint on "the thing kind of the individual concerned" in our speculation. Wiggins is somewhat convinced by this restriction, or at least treats it sympathetically. He writes

A man who entertains a contrary-to-fact speculation about Julius Caesar must leave himself room to rebut the charge that he has lost Caesar. This Roman consul need not have been a consul, need not have been a Roman, need

¹Ibid. p. 314.

²Op.cit. p. 334.

³Ibid. p. 335.

not have been male perhaps ..., but unless anything might be anything there must be some f such that, no matter what else he is, he can be picked upon as this f. And what else can f stand for here than the most general sortal specification of the object which is capable of individuating it fixing its existence or persistence condition and answering the question "what is it?"¹

Wiggins seems to want to rule out speculations based on the assumption that "anything might be anything". And yet we know that people have speculated about whether heat was a fluid, or whether meteorites were angels, or whether atoms were holes within holes. And even more interesting are speculations about whether matter can be transferred from place to place by becoming energy, or by some such process.

Speculation need not simply be counter-factual in kind. Just as interesting is speculation of the kind mentioned by Hintikka when he talks about 'possible courses of events'.² Future conditionals are most interesting here, especially when use is made of predicates which, in a sense, include identity. These are the predicates which do not assert that something is the same f as something else, quite the contrary. Such predicates are "became", or "changed into", or "changed his(its) form". These fall into a general category of predicates of identity through change, or identity through time. We do allow that something can change radically through time, for example, something's changing from matter to energy.

It could be said that in cases like an atomic particle's change into energy the particle is destroyed, that is, ceases to exist, and a new individual is created, or begins to exist. Two

¹Ibid. p. 335-336.

²In "Semantics for Propositional Attitudes", op. cit.

problems arise here. First, there is the problem of whether it can be said that anything changes at all. Second, there is the problem which arises, in speculation, about changes back into matter. For example, when someone speculates about matter transfer, can we say anything like, "The object is converted into energy which is transmitted over a distance and then changed back into the object"? It might be said that this is simply an example of an individual's ceasing to exist and then beginning to exist again. If this is the stance to be taken then we are back to the problem of continuity through radical change.

In the context of our modal logics we could take the possible worlds to be states of the world (or possible states of the world) at particular points of time. So, if the claim is then made that individuals can change radically from time to time, we can interpret this by saying that individuals can be radically different from world to world. Anything can become anything, but with one qualification which we now come to.

Matter and description

If we allow that individuals can be radically different from world to world then the individuals look very much like Aristotelian matter, and "matter only exists as somehow designate."¹ In our formal systems we do not allow individual constants to stand alone. Individual constants require predicates, and with a suitable predication we can show which individual is being designated. Sometimes this is done by means of a statement in which a definite description

¹G.E.M. Anscombe, "The Principle of Individuation", Proceedings of the Aristotelian Society Supplementary Vol. 27 (1953), p. 88.

is used, but not always. And just as an individual can change radically, or have radically differing predicates from world to world, so "Matter exists as somehow designate; but that is not enough to secure the permanent identifiability of a once designate bit of it."¹ In other words, we can provide in one world an identifying description of an individual, but there is no guarantee on that basis that any identifying description will be true of that individual across all possible worlds.

Searle has pointed out that definite descriptions do not designate analytically. Furthermore, as he also points out, although individuals can be designated by a complete description, the description is not a name, nor is it the sense of the name (or names) which also designate the individual. In terms of our theory this is certainly rules out, but it could be argued that a definite description could be used to determine the referent of the name and so the sense. But that is different. What Searle is getting at is that,

If we try to present a complete description of the object as the sense of a proper name, odd consequences would ensure, e.g. that any true statement about the object using the name as subject would be analytic, any false one self-contradictory, that the meaning of the name (and perhaps the identity of the object) would change every time there was any change at all in the object,...²

Searle contend that "though proper names do not normally assert or specify any characteristics, their referring uses nonetheless presuppose that the object to which they purport to refer has certain characteristics."³ Which characteristics is left open.

¹Ibid. p. 88.

²J. R. Searle, "Proper Names", Mind 67, 1958, p. 169.

³Ibid. p. 171.

This accords well with our understanding of individuals. As far as logical names are concerned, we are given a list of them, so there will be no confusion about whether or not some symbol (word) is a name or not, as there can be in a natural language. So whenever a name is mentioned we know that we have an expression which designates some individual, and we know that we can use the name in some formula in order to show which individual the name designates in terms of the properties the individual happens to have. But the only necessities determining the individuals having those properties are those which can be formalised in terms of consistent formulae. In other words an individual designated by a may have any property formalisable by Fa, but may never be described by Fa & \sim Fa.

It is this very last point which makes for the difficulties, or purported difficulties, about the systems $M^iQ = (1 \leq i \leq 4)$. At first sight it may seem as though any difficulty we had about an individual's having some set of properties when in one world but a completely different set in another could be discounted so long as we observe the rule of the indiscernibility of identicals within any world. But if we must observe this rule within any world, why should we not have to observe it across worlds? Otherwise it looks as if the necessities of consistency are arbitrarily suspended, or simply suspended.

But surely, that is, in a sense, what acceptance of an individual's changing radically must involve. It is not an arbitrary suspension of consistency, it is a recognition that if an individual changes then it has new properties and not the ones it had previously. Nevertheless, in a given context some account is usually expected of how the individual's properties have changed. The account need not

be a neat causal account or any rigorous account. With our time-traveller we inserted a time machine.¹ In a sense this is an attempt to provide a sense of continuity running through the speculation and fiction.² The supposition of continuity of individuals through change or from world to world seems to be fundamental. In as much as we understand what is said in contrary to fact statements or conjectures about the future or past we seem to be supposing such continuity or persistence of individuals. But always we must be clear about the context in which such speculations are made. What is allowable in physics will be more restricted than what is allowable in fiction. Anything can become anything, provided we can account for the changes (or differences).

Worlds and designations

The idea that anything can become anything is open to misinterpretation. For example, it might be suggested that Aristokles might have been Pericles and Pericles might have been Aristokles, since anything can be, or could have been, anything. And if Pericles were to have been Aristokles then Pericles' nickname would surely have been 'Plato'. So Pericles would have been Plato.

But, as has already been mentioned, even Quine treats identity in a somewhat different fashion to his treatment of other

¹Cf. a comment by I. Asimov in Where do we go from here? Book I, Sphere Books, London, 1974: "A scientific principle is deliberately bent for the sake of making a particular plot possible." p. 9.

²Cf. Thomason, op. cit. p. 138.

properties.¹ We are certainly treating identity in a fashion different to the way in which we are treating other predicates. Although I have argued against essentialism, I have also argued in favour of rigid designation. So, what is to be said about the sort of case outlined above where Pericles would have been Plato, and Aristokles Pericles?

In the first place we need to do some disentangling of the example. As set out above, the example can be quite confusing. The following is what is involved:

Pericles might have been called 'Aristokles'. Aristokles, with the broad shoulders, might have been called 'Pericles'..

One way of spelling this out in terms of possible worlds is to say that in world g (the real world) there is one individual called 'Pericles' and another called 'Aristokles', and that in world n (the world conjectured) the individual called 'Pericles' in g is called 'Aristocles', and the individual called 'Aristokles' in g is called 'Pericles'. If this is a description of what people are called in g and n, then we can say that Pericles is not Aristocles in world g nor is Pericles Aristokles in world n. But there seems to be a place for the following transworld identities

Pericles = Aristokles

Aristokles = Pericles

where the left sides of the identities are the names of individuals in g, and the right sides are the names of individuals in world n.

If we insist, then, on the theses that

¹Supra p. 84.

$$(A) \quad \underline{a} = \underline{b} \equiv \Box(a = b)$$

and

$$(B) \quad \underline{a} \neq \underline{b} \equiv \Box(\underline{a} \neq \underline{b}),$$

then it looks as though these theses and our example together with the transworld identities will lead to contradiction.

The main point we need to make is that although this example looks as though it will lead to contradiction, it will only do so if it is framed-up in a certain way formally. But if it is so framed-up there is a grave difficulty about the formalisation. This can be seen if, for $M^1Q =$, we let

' \underline{a}_1^n ' stand for 'Aristokles'

and ' \underline{b}_1^n ' stand for 'Pericles'.

Then we have:

$$(Ex)(Ey)(x \neq y \ \& \ x = \underline{a}_1^n \ \& \ y = \underline{b}_1^n \ \& \ \Diamond(x = \underline{b}_1^n \ \& \ y = \underline{a}_1^n)) \in \mu_n$$

i.e. the non-identical individuals which are called ' \underline{a}_1^n ' and ' \underline{b}_1^n ' in world \underline{n} are, in some possible world, called ' \underline{b}_1^n ' and ' \underline{a}_1^n ' respectively.

So it follows that

$$(\underline{c}_1^n \neq \underline{d}_1^n \ \& \ \underline{c}_1^n = \underline{a}_1^n \ \& \ \underline{d}_1^n = \underline{b}_1^n \ \& \ \Diamond(\underline{c}_1^n = \underline{b}_1^n \ \& \ \underline{d}_1^n = \underline{a}_1^n)) \in \mu_n$$

for some \underline{c}_1^n and some \underline{d}_1^n . So it follows that

$$(\underline{c}_1^n \neq \underline{d}_1^n \ \& \ \underline{c}_1^n = \underline{a}_1^n \ \& \ \underline{d}_1^n = \underline{b}_1^n \ \& \ \underline{c}_1^n = \underline{b}_1^n \ \& \ \underline{d}_1^n = \underline{a}_1^n) \in \mu_m$$

where μ_m is an alternative to μ_n . This will be contradictory. Lest someone object to formalizing

Aristokles might have been called 'Pericles'

as $\Diamond(\underline{a}_1^n = \underline{b}_1^n)$

we can simply set up the two model sets, in the one model system, both containing all the identities which are contained in the other and we will get the same contradiction. We begin with

$$(Ex)(Ey)(x \neq y \ \& \ x = \underline{a}_1^n \ \& \ y = \underline{b}_1^n) \in \mu_n,$$

then

$$(Ex)(Ey)(x = y \ \& \ x = a_1^n \ \& \ y = b_1^n) \in \mu_m,$$

then, to show that Aristokles in world \underline{n} is Pericles in world \underline{m} and that Pericles in world \underline{n} is Aristokles in world \underline{m} , we have

$$a_1^n = b_1^n \ \& \ b_1^n = a_1^n \in \mu_n \text{ and } \in \mu_m.$$

Even though this does lead to contradiction, and will also do so in N^2Q = so long as we use ' a_1^0 ' for 'Aristokles' and ' b_1^0 ' for 'Pericles', there is a problem which at once becomes obvious if we look at the last formula above.

This formula, $a_1^n = b_1^n \ \& \ b_1^n = a_1^n$, just does not convey the full sense of what it is supposed to convey. Intuitively, one would like to set out a symbolism like:

Let

' a_1^n ' stand for 'Aristokles (in world \underline{n})'

' b_1^n ' stand for 'Pericles (in world \underline{n})'

' a_1^m ' stand for 'Aristokles (in world \underline{m})'

' b_1^m ' stand for 'Pericles (in world \underline{m})'

Then we would have, instead of $a_1^n = b_1^n \ \& \ b_1^n = a_1^n$,

$$a_1^n = b_1^m \ \& \ b_1^n = a_1^m$$

as the transworld identity formula, and no contradiction would result. But, if we follow such intuitions we are saying that when 'Aristokles' is used as a name in world \underline{n} it is not necessarily the same name as 'Aristokles' used in world \underline{m} . The symbols ' a_1^n ' and ' a_1^m ' are quite different individual constants.

Kripke deals with this problem by pressing just this point. He discusses the possibility that 'Hesperus' and 'Phosphorus' might have been used to name two quite separate stars or planets. He says

it could have turned out that Hesperus was not Phosphorus; that is, in a counterfactual world in which 'Hesperus' and 'Phosphorus' were not used in the way that we use them, as names of this planet, but as names of some other objects,.... But we, using the names as we do right now, can say in advance, that if Hesperus and Phosphorus are one and the same then in no other possible world can they be different. We use 'Hesperus' as the name of a certain body and 'Phosphorus' as the name of a certain body. We use it as the name of those bodies in all possible worlds.¹

The emphasis here is on our use of the names in the real world. The real world has priority. The names rigidly designate what the names are in fact the names of.

There might be a possible world in which, a possible counterfactual situation in which, 'Hesperus' and 'Phosphorus' weren't names of the things they in fact are names of But still that's not a case in which Hesperus wasn't Phosphorus.²

Kripke's reasons for emphasising our use of the words in the real world are revealed in a footnote where he says,

Recall that we describe the situation in our language, not the language that the people in that [counterfactual] situation would have used. Hence we must use the terms Hesperus and Phosphorus with the same reference as in the actual world. The fact that people in that situation might or might not have used those names for different planets is irrelevant.³

It is not simply a matter of declaring that the referent of a name in the real world is its primary reference, or, in our terms, determines its sense. For Kripke, there is a sense in which the alternative uses suggested mean that a different language is being used by those who use the names not in the way in which we use them. The words, 'Aristokles' 'Pericles' 'Hesperus', 'Phosphorus',

¹Op. cit. p. 308.

²Ibid. p. 307.

³Ibid. fn 50 p. 350. cf. p. 289 last paragraph.

as words remain the same, but as names they are different.

The primacy of the actual world is also emphasised by Kripke when he argues that "it is because we can refer (rigidly) to Nixon, and stipulate that we are speaking of what might have happened to him (under certain circumstances), that 'transworld identifications' are unproblematic in such cases."¹ "We can refer to the object and ask what might have happened to it. So, we do not begin with worlds ... and then ask about criteria of transworld identification; on the contrary, we begin with the objects, which we have, and can identify, in the actual world."²

Apart from the difficulty of accepting the notion that, in the world where 'Aristokles' is the name of the person we call 'Pericles' and 'Pericles' is the name of the person we call 'Aristokles', there is a different language being spoken (perhaps a dialect of English), there is also the question of whether the names of fictional or imaginary individuals are rigid designators for Kripke.

The latter question could be answered in terms of the primacy of the world in which the individual named is identified as such. Roughly put, we can say that a name like "Pegasus" is a rigid designator, but in order to identify the individual referred to by "Pegasus" we should consider first the world of Greek mythology. That world is, as Kripke says, a stipulated world. Once we know what was stipulated of Pegasus in the primary world of Pegasus' existence then we can treat "Pegasus" as a rigid designator. This means that possible worlds are virtually indexed. One will be the real world, another the

¹Ibid. p. 270.

²Ibid. p. 273.

world of Greek mythology, another the world of some counterfactual conjecture, and so on.

Rennie appropriates the symbol ' a_k ' as a constant for the real world and the symbol ' a_k ' for the tense-logical world of the present. In this way the worlds can be indexed. Our use of superscripts on individual constants could be seen as a way of indicating the world in which the constant is established as a rigid designator. The superscript o on individual constants in $M^2Q =$ will not be of any real use in indicating the world of primary use. But in $M^3Q =$ and $M^2Q =$ (with the exception mentioned) the superscripts could be most helpful. We have already indicated how one might set out a dictionary for ' a_1^n ', ' a_1^m ', ' b_1^n ' and ' b_1^m ', to capture the full sense of the Pericles-Aristokles case.

Finally, we return to the question of Napoleon and Hannibal's son. If we accept the idea that the real world has priority and that some account should be given of changes from world to world, then the flat declaration that Hannibal called his son "Napoleon" in the world of conjecture will just not do as an account of how Hannibal's son might be the Emperor Napoleon. We have just argued that what some one might have been called is secondary both to what they were called in fact and also to how the name in question is used in fact. It is a question of giving names. Of course, some inventive writer might give an account of how the Emperor Napoleon did such and such and became Hannibal's son. For the purposes of that account we might accept that Hannibal's son was Napoleon, but not on the basis of an account which says only that Hannibal called his son "Napoleon."

Language and essentialism

When we turn to the question of a difference of language from world to world we will find that Kripke is strongly committed to this view. It will not do for us simply to say that it is the same language but with some alterations in nomenclature. In the first place, those alterations in naming might not affect the syntax of the language but they must affect the semantics. Truth conditions will change from world to world if we insist that the semantics must deal with words as words, or even if we insist that the same names are being used. But also, for Kripke, not only can a change in nomenclature mean a change in naming, it can also mean a change in predicating. We can not only say that Aristokles might have been called "Pericles", but we can say that the property of being a philosopher might have been called the property of "being a democratic leader". So, a change in nomenclature could most certainly mean a change in language. In some possible world the sentence "Pericles was a democratic leader" would mean that Aristokles was a philosopher.

Even if we say that, in the world where "Pericles was a democratic leader" means that Aristokles was a philosopher, English is simply being used in a non-standard way, non-standard English is not standard English. "In describing that world, we use English with our meanings and our references."

This leads us on to make the point that in $M^3Q =$, as opposed to $M^1Q =$, when we have

$$(Ex)(Ey)(x \neq y \ \& \ x = a_1^n \ \& \ y = b_1^n) \in \mu_i,$$

the existential instantiation will use individual constants of the superscript i to give, for example,

$$c_1^i \neq d_1^i \ \& \ c_1^i = a_1^n \ \& \ d_1^i = b_1^n \in \mu_i.$$

The quantifiers, being world relative and not in the scope of any modal operator, bind the variables which range over the individuals in world i . ' c_1^i ' and ' d_1^i ' rigidly designate in world i and so make ' a_1^n ' and ' b_1^n ' rigidly designate individuals in world i .

The individual named by ' a_1^n ' and ' c_1^i ' may or may not exist in any other world than world i (i is not necessarily distinct from u), but ' a_1^n ' and ' c_1^i ' are rigid designators nonetheless. So we could say of world m that a_1^n does not exist:

$$\sim (Ex)(x = a_1^n) \in \mu_m$$

Yet even here ' a_1^n ' refers to a_1^n .

If we are to have any essentialism then it can only be, from a logical point of view, an essentialism based on the consistent use of language. Names should always (in all possible worlds) designate the same individuals and predicates should not change meaning.

CHAPTER FIVE

FREE LOGIC

In the first chapter of this essay a question was asked about whether it is possible to consider situations in which named and existent individuals no longer existed. In the third chapter of this essay we, in fact, did consider such situations. In the logical systems $M^iQ = (2 \leq i \leq 4)$ we considered a variety of situations where named individuals existed in but one world and did not exist in another. We now turn in more detail to logics which take account of the fact that some things do not exist. Such logics are known as existential presupposition free logics, or simply free logics.

Free logics were constructed on the assumption that there could be logical names which did not designate existing individuals. This assumption is quite contrary to Russell's and Quine's assumptions. We shall turn, in the next section, to a consideration of Quine's theories in this area. But before we do we should note that the notion of rigid designation does not involve the assumption that what is designated must exist. A rigid designator has the same reference in all possible worlds, says Kripke, but

I also don't mean to imply that the thing exists in all possible worlds, just that the name refers rigidly to that thing. If you say 'suppose Hitler had never been born' then 'Hitler' refers, here, still rigidly, to something that would not exist in the counterfactual situation described.¹

Furthermore, Kripke introduces the notion of a strongly rigid designator, which is the designator of something which is

¹Op. cit. p. 290.

necessarily existent.¹

Empty Singular Terms

It seems to be one of Quine's main contentions that the use of names to refer to non-existing things, such as Pegasus, results in complete confusion. He has two lines of argument. One is concerned with quantification, the other with truth values.

Quine's arguments about names and quantifiers and things that don't exist are marked by a tortuous avoidance of the expression "exists" and its cognates together with the use of "there is" to mean both "there exists" and "there is at least one."

In the essay "On What there is"² Quine uses 'exists' and its cognates in only one paragraph. In that paragraph he says that since a group of philosophers have ruined the word 'exist' "I'll try not to use it again: I still have 'is'."³ He then ruins the word 'is'.

In Word and Object he links existence and quantification together in a very strong way. He writes "there is little evident sense in ' $(\underline{x})(\underline{x} \text{ exists})$ ' or ' $(\exists \underline{x})(\underline{x} \text{ exists})$ '. A look at ' $(\exists \underline{x})(\underline{x} \text{ exists})$ ' suggests that our embarrassment may be one of riches: that 'exists' has perhaps no independent business in our vocabulary when ' $(\exists x)$ ' is at our disposal."⁴ Later he goes on: "Such is simply

¹Ibid. p. 270.

²W.V.O. Quine, From a logical point of view 2nd Ed. Harvard University Press, Massachusetts (1964).

³Ibid. p. 3.

⁴Op.cit. p. 176.

the intended sense of the quantifiers ' (\underline{x}) ' and ' $(\exists \underline{x})$ ': 'every object \underline{x} is such that,' 'there is an object \underline{x} such that.'"¹

But perhaps we should neither be embarrassed, nor fail to see some sense in ' $(\exists \underline{x})(\underline{x}$ exists)'. For example, just as ' $(\exists \underline{x})(\underline{x}$ exists)' can be read as, 'something exists' so we can read ' $\sim(\exists \underline{x})(\underline{x}$ exists)' as 'something does not exist' (e.g. Pegasus). The distinction between these last two readings will be lost if we get embarrassed.

Furthermore, Quine's intended readings of the quantifiers show that he packs more into the quantifiers than pure matters of quantity. By doing this he is able to misread other writers and say it is their confusion. He writes, "We find philosophers allowing themselves not only abstract terms but even pretty unmistakable quantifications over abstract objects ("There are concepts with which," "...some of which propositions," "...there is something that he doubts or believes"), and still blandly disavowing, with the paragraph, any claim that there are such objects. Pressed, they may explain that abstract objects do not exist the way physical ones do."² (my italics) It is clear here that Quine slides from "There are ϕ " to " ϕ exists." He goes on to say that unless we make such a slide we must hold that there are at least two senses for "there are", one for physical objects and one for abstract ones. But why?

We could just as well say that "there are" is purely quantificational. We can say "There are objects", or "there are concepts", or "there are fictional characters" and be merely saying

¹Ibid. p. 242.

²Ibid. p. 241.

that we can list at least one of each. Hence ' $(\exists x)(\sim x \text{ exists} \ \& \ x = \text{Pegasus})$ ' could be read as 'Pegasus can be entered on the list of things which do not exist.' In case the word 'things' causes trouble we can take it in the same sense as it has in "Some things do not exist", or we could read the formula as 'Pegasus can be entered on the list as not existing'.

There is, it seems to me, a purely quantificational interpretation which can be given to quantifiers. In the standard set theoretic semantics we interpret this purely quantificational sense in terms of a non-empty set of objects. We consider what quantifiers involve, not in terms of the kinds of objects we put in the set, but in terms of how all or at least one relate.

But generally, it is more to the point to include in the sense of the quantifier the abstraction predicate which establishes the membership of the non-empty set. For Quine, this means the kind of interpretation he gives. Then ' $(\exists y)(y = \text{Pegasus})$ ' can be read as 'At least one existing thing is Pegasus' just as Quine suggested. Similarly, ' $(x)(\exists y)(x = y)$ ' reads as 'every existing thing is such that it is something which exists.'

If we introduce additional quantifiers, such as ' $(\forall x)$ ' and ' (Σx) ', as purely quantitative, then ' $(\forall x)(\exists y)(x = y)$ ' reads as 'everything exists', and ' $(\Sigma x) \sim (\exists y)(x = y)$ ' reads as 'Something does not exist.'¹

Free logics usually begin by giving the standard quantifiers the additional Quinean strength. In these logics the quantifiers not only indicate quantity but also predicate existence of those individuals over which the bound variables range. A certain analogy

¹R. Routley, "Some things do not exist", Notre Dame Journal of Formal Logic (7) 1966

can be seen here between such a free logic and the systems set out above. ($M^iQ = (1 \leq i \leq 4)$). This analogy can break down at a point which we discuss later.

If we took a model system in which one model set described the world of existing individuals then the bound variables in that model set would range over existing individuals and the names occurring (except in identities) would designate existing individuals. In every other model set the names used (except in identities) would designate non-existing individuals in any one of several domains of non-existing individuals.

In the systems set out above we have the additional complication that we have, in terms of our analogy, several domains of non-existing individuals together with quantification over these domains. This complication is set out in a different fashion in each $M^iQ = (1 \leq i \leq 4)$.

Free logics have also been made more complicated by various writers. The first move, as mentioned above, is to introduce an additional quantifier to range over both domains. Formulae of the form " $(Ex) \phi$ " are then read as "There exists at least one x such that ϕ ", whilst formulae of the form " $(\Sigma x) \phi$ " are read as "There is at least one possible or existing x such that ϕ ."

The second move is to allow the free variables to designate impossible objects. This move means that we introduce a third domain of individuals, a domain of impossible individuals. Attempts have been made to introduce a quantifier to range over all three domains. This has met with only limited success.

Free Modal Logic

In "Modal Logic and Metaphysics",¹ as we have already noted,² Thomason develops a free modal logic called 'Q3'. The relationship between the sets of individuals in each world is analogous to our M^2QI . There are three main differences between Thomason's Q3 and our M^2QI . The first is that, whereas in M^2QI , we assume that there is at least one individual common to all worlds and perhaps denumerably many, in Q3 it is possible for there to be no individuals common to all worlds. In this way Q3 is more like M^3QI than M^2QI . The second is that the quantifiers have as their domain those individuals which are common to all worlds. The individual variables range over existing individuals only, and the existing individuals are the substances only. As Thomason says,

We will construe individual variables as ranging over substances, and thus take seriously the classical doctrine that only substances are "beings" in the fullest sense of the word.³

This means that the quantifiers, in any one model system for Q3, will all have the same domain of quantification from world to world. The third difference is that the definite descriptions are not empty as in M^2QI . What are commonly called 'empty definite descriptions' are given values which are non-substances. This is like saying that definite descriptions in M^2QI (or M^3QI) can designate individuals not common to all worlds, or individuals which occur in one world only.

So descriptions can designate individuals without

¹Op.cit. pp. 138 ff.

²Supra p. 96.

³Op.cit. p. 138-139.

designating the values of variables. When we come to names such as "Socrates" or "Texas", Thomason at first vacillates, but eventually decides to take them as he does definite descriptions. He says

The question of how to handle individual constants is delicate. Most uses of proper names seem to be of the sort in which substances are intended; e.g.

$(x)(x = \text{Socrates} \supset \Box x = \text{Socrates})$

and

$(x)(x = \text{Texas} \supset \Box x = \text{Texas})$

have a ring of truth. This would suggest that in Q3 individual constants should be treated like individual variables. There are, however, some uses of proper names (roughly classifiable as titulary uses) which do not meet the condition. For example, if we call anyone holding a certain political office "Caesar", then various people may be Caesar in different possible worlds, and

$(x)(x = \text{Caesar} \supset \Box x = \text{Caesar})$

or equivalently,

$E \Box \text{Caesar}^*$

is false It seems rather artificial to make a syntactical distinction between these and other sorts of namings, and therefore in Q3 I will treat individual constants more like descriptions than individual variables.... The supposition that an individual constant refers to a substance can, however, always be made explicitly, by means of assertions of the sort

$E \Box \underline{a}^1$

This means that names can be used to refer to non-existent individuals, so empty singular terms in general can be given values in sets of non-existent individuals. This also means that individual constants need not be rigid designators. Just as definite descriptions can refer to a different individual from world to world, so can Thomason's names. This becomes quite clear when he gives a counterexample to

$(x) \Box Ex \supset (Ea \supset \Box Ea)$

In the counterexample the value given to 'a' is different from world

¹Ibid. p. 139-140.

* This is defined by Thomason as $(\exists x) \Box (x = \text{Caesar})$ i.e. 'Something is, in all worlds, Caesar', or, 'Caesar is a substance'. Also $Ea =_{df} (\exists x)(x = \underline{a})$

to world.¹

In all our logics we have assumed that all the individuals in each world exist in that world. So our individual constants have designated existing individuals. Definite descriptions have not always done so. If we wanted to make our logics into free modal logics then we have open to us certain alternatives. We could, on the one hand, construct a logic similar to Q3, or on the other hand, we could simply extend our present systems. The first difference between these alternatives is that for the former we adopt a policy of having just one set of existing individuals for all worlds, whether empty or not, but for the latter we simply have in each world a set of existing and a set of non-existing individuals without assuming that the individuals which exist in some arbitrary world n will all also exist in any other world. The second difference is that for the former we would have to consider seriously the policy of taking names to be rigid designators with a view to dropping it, whereas for the extending of our own systems we would still operate on the presupposition of rigid designation.

Taking up the second area of difference first, I would want to argue against the abandonment of the principle of names' being assumed to be rigid designators. The reasons already set out in favour of names' being assumed to be rigid designators will make up part of the case. The other part of the case is that names should not be treated exactly like definite descriptions. Thomason's final move in the long passage quoted above is not simply to let names designate non-existent individuals like definite descriptions can, but to treat names as though they are non-rigid designators.

¹Ibid. p. 140.

Much of Thomason's argument here depends on the example he uses -- Caesar. Although he begins with 'Socrates' and 'Texas', which are names, the example of Caesar is one of title. Now it seems methodologically wrong to treat all names in a certain way because some are exceptional rather than in another way which most would indicate. In particular, since the exceptional names are titles it seems even more unsound. Many titles are clearly definite descriptions, and especially so in their full form. "The Queen" could be short for "The Queen of Great Britain, Australia, Canada, New Zealand and the other Dominions and territories, Defender of the Faith," but it could also be short for the full title of the Queen of the Netherlands, or the Queen of Denmark. On this point I should rather take constants to be rigid designators and deal with titles separately.

As to the first area of difference, we have already seen that we are looking for a logic in which we can express the idea that individuals existing in one possible world do not exist in another. So it seems best to extend our own systems.

Before constructing any formal system we once again follow the procedure of setting out a policy to guide our moves. In this case we need to be clear about the role of quantifiers in the logics proposed. As the systems $M^iQ = (1 \leq i \leq 4)$ stand at the moment the quantifiers combine both the particular-universal quantity role and the existential role. If in the free logics we construct the individuals in each world will be in just one of two disjoint sets, one an inner domain of existing individuals for that world and the other an outer domain of non-existing objects for that world, then we must decide whether the present symbols, 'U' and 'E', are to be the quantity quantifiers or to be the existential presupposition quantifiers. For

the sake of uniformity with Hintikka's work¹ we will use 'U' and 'E' as the quantifiers for domains of existing objects. We will introduce ' Π ' and ' Σ ' to replace 'U' and 'E' in the present rules.

So we begin to extend $M^iQ = (1 \leq i \leq 4)$ by replacing all occurrences of 'U' by ' Π ' in all the sets of rules C_M^i ($1 \leq i \leq 4$) to give C_F^i respectively. Needless to say, we add ' Π ' and ' Σ ' to the improper symbols for each system. Appropriate formation rules, analogous to the rules for quantified formulae with 'U' and 'E', are also added. These will be found in Appendix II. The extended systems are M^iQF ($1 \leq i \leq 4$).

We are now faced with the problem of constructing rules for formulae of the forms $(U\bar{x})A$ and $(E\bar{x})A$. Fortunately the work has already been done for us, so we can borrow the rules from Hintikka's work.² Some modification is needed, but not much. We have to allow for the various restrictions based on the superscripting of individual constants. We begin with rules to be added to C_F^1 to give C_F^1 :

- (C.E₀) If $(E\bar{x})A \in \mu_n \in \Omega$ then $(E\bar{x})(x = a) \in \mu_n$ and $(A(a/\bar{x})) \in \mu_n$ for some individual constant a .
- (C.U₀) If $(U\bar{x})A \in \mu_n \in \Omega$ and if $(E\bar{x})(x = a) \in \mu_n$ then $(A(a/\bar{x})) \in \mu_n$.
- (C.E Σ) If $(E\bar{x})A \in \mu_n \in \Omega$ then $(\Sigma \bar{x})A \in \mu_n$.
- (C. Π U) If $(\Pi \bar{x})A \in \mu_n \in \Omega$ then $(U\bar{x})A \in \mu_n$.

The first rule simply indicates that if some existing thing is such that A then something, namely a , exists and a is such that A . The second rule indicates that we can only universally instantiate formulae of the form $(U\bar{x})A$ to values in the set of existing individuals.

¹Especially in Knowledge and Belief.

²Ibid. pp. 126-131.

The third rule indicates that if at least one existing thing is such that \underline{A} then at least one thing is such that A . The fourth rule indicates that if everything is such that \underline{A} then every existing thing will be such that \underline{A} .

Next, the rules to be added to C_{F*}^2 to give C_F^2 :

(C.E Σ) as above, and (C. Π U) as above, and

(C.E Σ^0) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then either $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$ and $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$, or $(E\underline{x})(\underline{x} = \underline{a}^0) \in \mu_n$ and $(\underline{A}(\underline{a}^0/\underline{x})) \in \mu_n$, for some individual constant \underline{a}^n or \underline{a}^0 .

(C.U 0) If $(U\underline{x})\underline{A} \in \mu_n \in \Omega$ and if $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$ then $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$.

The rules to be added to both C_{F*}^3 and C_{F*}^4 to give, respectively, the sets C_F^3 and C_F^4 are (C.E Σ), (C. Π U) and (C.U 0) and also:

(C.E 0) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$ and $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$ for some individual constant \underline{a}^n .

All the systems $M^{iQF}(1 \leq i \leq 3)$ are similar to Q3 in that there is no presupposition that an individual which does not exist in one world has to exist in at least one other possible world. So it is possible to say of some individual, \underline{a} , that it does not exist in any possible world. We have already seen that, in Q3, we can say that an individual exists in all possible worlds. The same can be said in any of $M^{iQF}(1 \leq i \leq 3)$.

The systems are also similar in that the formulae

$$(U\underline{x}) \Diamond \sim (E\underline{y})(\underline{y} = \underline{x})$$

and $(U\underline{x}) \Box \Diamond \sim (E\underline{y})(\underline{y} = \underline{x})$

are satisfiable (not self-sustaining). As Thomason says, "It therefore is consistent in Q3 to suppose that everything is perishable, or even

that everything is necessarily perishable."¹

The values of variables

In our discussion so far we have relied, very informally, upon the standard kind of set theoretic semantics in order to interpret our formal languages. The fact that such semantics postulate domains of individuals can lead to the assumption that these individuals are somehow "in the world" or somehow "real". This assumption can then lead further to the consideration of just what these individuals are. Various kinds of being are then attributed to these somehow "real" individuals.

Some logicians simply refer to these individuals as the values² of the constants. The domain of non-existent individuals in any world turns out to be the domain of values for empty singular terms. Real objects, or substances, or atoms, or some such things are the members of the domain of existing individuals. What then is a value which is not a real object, or a substance?

One response to this question is simply to say that the question is pointless. Empty singular terms are terms "purporting to refer to a well-defined object"³ but for which in fact there is no referent. If we give such a term a value then we do no more than acknowledge that it purports to refer. That is what the value amounts to in our formal semantics. Empty names are used in language, and the users are well aware that there do not exist any objects which are, in fact, the referents of those names. The names are used as

¹Op. cit. p. 143.

²Cf. Thomason, op.cit. p. 130.

³J. Hintikka, Knowledge and Belief, Cornell University Press, Ithaca, 1962. p. 126.

names and, although they are not given to objects which exist, criteria for the identification of that to which a name is given can be stipulated. "Pegasus" is the name of a winged horse which does not exist. So we give "Pegasus" a value in a domain of values for non-existent items. The value of an empty singular term is an object which does not exist.

This response is, in a sense, the putting off of the problem. The response is that since in our language we can speak of objects, some of which exist and some of which do not, in our semantics we will simply provide a set theoretic framework to reflect this feature of language. We will have a set of things said to exist, and a set of things said not to exist. Problems about what it is for something to exist or not to exist can be left aside for philosophical discussion.

Another response to the problem of the values of empty singular terms is the response given by those who interpret quantification in a substitutionary fashion rather than in an objectual fashion. The difference between these two interpretations of quantifiers and the variables they bind is the subject of much debate. According to a substitutional interpretation a universal quantification $(\forall x)A$ of a first-order language is true if every replacement of x everywhere in A by an individual constant is true, an existential one $(\exists x)A$ true if some replacement of x everywhere in A by an individual constant is true.¹ According to the objectual interpretation a universal quantification $(\forall x)A$ is true if every object in the domain of the quantifier is such that A , an existential one $(\exists x)A$ true if some object in the domain is such that A .

The two kinds of interpretation are often contrasted in terms of the values of the individual variables. It is said that the

¹This sentence is almost word for word from H. Leblanc "Truth-value Semantics for a Logic of Existence", Notre Dame Journal of Formal Logic, Vol. 15, 1974, p. 153.

individual variables, from a substitutional point of view, are taken as placeholders for individual constants. So $(\exists x)A$ is true if the place(s) held by x in A can hold some constant and so yield a true instance. At least one of $(A(a/x))$, $(A(b/x))$, $(A(c/x))$, ... will have to be true for $(\exists x)A$ to be true. By contrast, it is said that, from an objectual point of view, individual variables refer to objects of some sort.

Those who respond in terms of a substitutionary interpretation of quantifiers will say that we do not need to consider what it is to which any singular term refers. We only need to know that such terms are what they are, and that they purport to refer. Questions about what they refer to are questions of ontology. Logical systems are set up basically to provide us with a means of deciding on the validity of arguments.

Leblanc, supporting substitutional interpretation, writes,

Some like their logic mixed with a lot of ontology. To them first-order logic, for example, adjudicates on basic matters of existence, and any semantic account of it should come with things, sets of things, and relations between things

Others prefer their logic straight: they view it as a handbook (of a highly sophisticated kind, to be sure) for drawing inferences. Of course, there are things, and if one is to discourse about them, he must -- sooner or later -- think of them as belonging to sets bearing relations to one another, and so on. But first-order logic can be, and hence is perhaps best, explicated without recourse to "models": What is dispensable is not of the essence.¹

Since we have been making use of Hintikka's model set model system semantics, and will make use of his epistemic logic in another Chapter, we will now turn to the question of the interpretation

¹H. Leblanc, "On Dispensing with Things and Worlds" in Logic and Ontology, Ed. M.K. Munitz, New York University Press, New York, 1973, p. 241.

of quantifiers in his logic. In the light of such a discussion we should be able to arrive at some conclusion about the response given to the question raised about individuals -- the response that a substitutionary interpretation of quantifiers will solve the problem. The solution is, of course, to remove the question from logic and place it in ontology.

The range of quantifiers.

Hintikka has argued that quantification into opaque contexts can be interpreted in such a way as to make sense. As a matter of fact, Hintikka's semantics does make sense if the values of the variables are taken to be expressions, i.e. singular terms that are substituted for the variables. However, Hintikka does not intend to give this kind of 'substitutional' interpretation of quantification. He wants, and I think rightly, the values of variables to be "real, fullfledged individuals."¹

Or, as Hintikka himself writes,

At least one reader of KB (Knowledge and Belief) has got the impression that I am there relying on some sort of unconventional interpretation of quantifiers in terms of substitution-instances of quantified sentences rather than the normal interpretation according to which bound variables range over genuine individuals of some suitable sort ... I am on the contrary relying heavily on the idea that the values of bound variables have to be real, fullfledged individuals -- which seems to me the only way of making satisfactory sense of quantification.²

Substitutional Interpretation

Although Hintikka is quite definite about the way quantifiers should be interpreted, let us consider first whether the kind of semantical model in Knowledge and Belief is open to either a

¹Dagfinn Føllesdal: "Quine on Modality" Synthese, Vol. 19 No. 1/2, December 1968, p. 150.

²J. J. Hintikka, "Individuals, Possible Worlds, and Epistemic Logic", Nous, Vol. 1 No. 1 p. 38.

substitutional or a standard set theoretic type of interpretation, and secondly, if the semantical model is open to either, then why Hintikka thinks it's so important to interpret it in his way.

The following passages from Knowledge and Belief are crucial:

The introduction of our symbolic notation could be reinterpreted as the introduction of the notion of a formula. The relation of formulas and the sentences we are studying will then be such that the latter are substitution instances of the former. (Sentences may be substituted for atomic formulas, names for free individual symbols, and so on.) From this point of view, we can conveniently specify the sentences we are interested in as substitution-instances of the formulas ...¹

We recognize the possibility that some of our atomic (and a fortiori nonatomic) sentences may contain names and other singular terms each purporting to refer to a well-defined object. Such terms will be represented in our symbolic notation by free individual symbols.²

How are the existential presuppositions to be avoided? We want to restrict the applicability of (C.U) to individual symbols h which are not empty. In other words, we want to make the applicability of (C.U) contingent on the presence of the sentence 'h exists' in μ .³

These passages not only make it clear that we can interpret the variables substitutionally, but that this seems to be indicated. The first quotation seems to say as much. But the third is even more decisive.

The third quotation shows us that in order to provide an existence presupposition free system we are to make (C.U)'s applicability contingent, not on whether or not h refers to or designates something existing, but on whether or not there occurs a sentence "h exists" in a set of sentences. So, whatever name

¹Op.cit. p. 11.

²Ibid. p. 126.

³Ibid. p. 129. The rule (C.U) referred to is virtually the same as our rule (C.U) (see Appendix II p.).

may be truly or defensibly substituted for x in p when we have the sentence $(Ux)p(x/h)$. This is what the rule $(C.U_0)$ can be seen as saying in both Hintikka's logic and our own above.

Furthermore, it is not readily obvious how one would give the semantical model in Knowledge and Belief the kind of interpretation which Hintikka wants. Purely prima facie, the system deals with sets of expressions, not with domains of individuals. We are not concerned about whether h is in the domain of existing individuals or not, we are concerned about whether the sentence " h exists" is in the set of sentences which we are given to deal with.

Even so, in Hintikka's preferred interpretation, the union of the two sets will be "the totality of objects which our language speaks of."¹ The set of existent individuals will be the set of existing objects in "the actual world".²

For Hintikka, of the objects of which our language speaks, some can be referred to in actuality, others cannot. Bound variables range over the former.³ Yet, when we look at the rules in Knowledge and Belief* it is easy to read them as saying that, of the objects of which our language speaks, some are spoken of as existing, and others as not existing. For bound variables, we can, with the relevant restrictions according to the quantifier, substitute defensibly the name of an object which is said to exist.

¹"Semantics for Propositional Attitudes", p. 23.

²Ibid. p. 23.

³In "Semantics for Propositional Attitudes" Hintikka gives the more formal rules which will enable us to interpret the system in Knowledge and Belief in the way just indicated.

* Hereafter we refer to Hintikka's epistemic logic as 'HS4'. The rules are found in Appendix III.

At least on first sight there seems to be as much warrant for interpreting the quantifiers in HS4 substitutionally as in any other way. But there is a general consideration that adds weight to this.

HS4 has a semantics which tell us which formulae of the system are defensible (satisfiable) in terms of sets of formulae, not in terms of sets of individuals. So it seems that HS4 formalizes what can defensibly be said. If HS4 were to be understood as a strict way of speaking truthfully about objects in the world and imagined (or possible) objects, including people, in terms of extensional predicates and propositional attitudes, then surely a set theoretic semantic model would have been more helpful.

Reference

Nevertheless, Hintikka insists upon the non-substitutionary interpretation. This turns out to be important because Hintikka is attempting to establish a referential theory of meaning for first order languages. This he is trying to do by incorporating a referential aspect into the interpretation of the quantifier.

But it is not clear just why, if one holds to a referential theory of meaning, one should resist a substitutionary interpretation of quantifiers. Nor is it clear that a set theoretic semantics must be that much more referential.

There seems to be no reason why the kinds of rules Hintikka gives in "Semantics for Propositional Attitudes" cannot be used to apply the semantical model of HS4 to the world. Given some model system for HS4 in which all the quantifiers have been resolved into the appropriate instances we still have to settle the ontological

questions about what individual constants name. Given the formula $(\forall x)(x \neq b)$, which is one way of formalizing "b does not exist", and resolving it into a series of instantiations, we have not somehow rid ourselves of the problem of what 'b' designates. We have to come back to ontology at some point.

Set theoretic semantics, using set theoretic models which Leblanc so dislikes, are not by any means adjudicators or ontological questions. There are set theoretic semantics in which the domains of individuals are explicitly not domains of real objects in any meaningful ontological sense. Certainly they are not the objects which we suppose exist. For example, Hughes and Cresswell, in their semantics for first-order quantification theory, use as the domain of objects

the set of all the individual-variables themselves,
considered as objects (letters, typographical entities).¹

It can hardly be claimed that this is an objectual interpretation in any ontologically meaningful sense.

So far, then, it seems as though neither the substitutionary nor the objectual interpretation of quantifiers will really determine what empty singular terms take as values in free logic. But in discussing these two views one thing emerges. Atomic formulae, containing no quantifier, are of crucial importance. It is to them that we make basic assignments of truth-value. So we need to consider the various options open to us in the assigning of truth-value to formulae containing empty singular terms. Instead of asking "to what do empty singular terms refer?", or "what are their values?", we ask "Since empty singular terms have no referents, on what basis do we assign truth-value to statements in which such terms occur?"

¹Op.cit. p. 161.

Truth Value Gaps

In the simpler (non-modal) existential presupposition-free logics, such as those set out by Leblanc and Thomason,¹ various options are open as regards the truth of statements containing a constant which does not designate any value of a variable or does designate something not a value of a variable. Leblanc and Thomason set them out as:

- (i) the statements in question may be denied any truth-value whatsoever; (ii) they may all be assigned some truth value other than the classical \underline{T} and \underline{F} ; (iii) some may be assigned \underline{T} or \underline{F} , and the rest assigned no truth-value; (iv) they may all be assigned the truth-value \underline{T} ; (v) they may all be assigned the truth-value \underline{F} ; and (vi) some (but not all) may be assigned \underline{T} , and the rest \underline{F} (vi'), in which all atomic statements containing a non-designating constant are arbitrarily assigned one of the two truth-values \underline{T} and \underline{F} , and the truth value of non-atomic ones is determined by the standard semantical rules of truth.²

For Quine, in a sense, option (i) applies. He writes, "for us who know that there is no such thing as Pegasus, the sentence 'Pegasus flies' counts perhaps as neither true nor false."³ That is, if we allowed "Pegasus" as a name, then option (i) would apply, so Quine contends, and since option (i) is a "truth-value gap" option and truth value gaps are undesirable when a sentence is not an open sentence, then if we want to avoid the undesirable consequence we must not allow "Pegasus" as a name.

¹H. Leblanc and R. Thomason, "Completeness Theorems for Some Presupposition-free Logics", Fundamenta Mathematicae, 62, (1968), pp. 123-164.

²Ibid. p. 126.

³Quine, Word and Object, p. 176.

But, not only is (i) not the only option, most of us would say that "Pegasus flies" is true. And if "Pegasus flies" is not true in a plain bald sense of 'true', then at least it is true-in-mythology. It is that simply because the name "Pegasus" was stipulatively given to the winged horse on which Bellerophon rode. We might not be able to say whether Pegasus' hooves are gold or black or white, but we can say that Pegasus flies. So, at least in some cases, we would be inclined to drop option (i). Option (ii) could give us the possibility of introducing the value M, mythically true, or even, true in some fictional story. Perhaps we could introduce an option (ii'): the statements in question could be assigned any one of several truth-values other than the classical T or F. Under (iii) we could introduce M for mythically true and A for mythically false, and so assign M to "Pegasus flies" and A to "Morpheus is the god of sleep."

Leblanc and Thomason concern themselves with (iv)(v) and (vi'). (iv) could be justified in a weak sort of way by simply saying that anything is possible of fictional or possible objects. But this option would allow us to affirm truly of Pegasus, for example, that he flies and does not fly. That is just absurd. Conversely, we could introduce modalities into the assignment of truth values to statements containing non-designating constants, and so say that it is possibly true that Pegasus flies and possibly false. But such a move would be more in line with option (ii) than (iv).

Option (v) is a kind of Russellian option. Were he to allow such things as non-designating constants it seems likely that he would count all statements containing them as false. But here again, if we allow non-designating constants then surely statements such as "Pegasus flies" or "Pickwick is fat" or "Excalibur is invincible"

should be counted as true.

Option (vi') seems to be a reasonably agreeable one. But it does suffer from the deficiency that it is analogous to some $M^iQ =$ with only two possible worlds, one being the real world. This deficiency means that what is said about fictional or possible objects must be consistent in the standard truth functional sense. So we could not use a formal system which reflects (vi') to rationalise, for example, a set of three books, each of which is consistent in itself, one being an accurate history, one being a fictional story set in that same historical period, and the other being a story with the same characters but doing different things.

In a free logic we could rationalise the example if we gave distinct logical names to all the characters. For example, if the King featured in each book, we could designate him in the history by k_1 , in the first work of fiction by k_2 , and in the second by k_3 . It would not be open to us to assert ' $k_1 = k_2$ ' because of the differing predicates.

But in the systems $(M^iQ = (1 \leq i \leq 3))$ we could rationalise the three books and have ' k_1 ' designate the King in each. This is the point at which the analogy between free logic and the systems $M^iQ = (1 \leq i \leq 4)$ can break down. The analogy can be restored if we take up the option set out as (ii'). What we do is introduce the truth values ' \underline{M} ' and ' $\sim \underline{M}$ ' to be 'possibly true' and 'not possibly true' respectively. In other words, we have modalised free logic.

CHAPTER SIX

KNOWING, REFERRING AND QUANTIFYING

One of the problems with which we began this essay was Frege's problem about the difference in cognitive value between 'a = a' and 'a = b' when a and b are the same entity. Frege's problem can be seen as a problem about the difference in truth conditions between

a knows that b = b

a knows that b = c

when b = c (as it must be, when the second formula above is true).

If we frame Frege's problem in this way then any further discussion will involve both the logic of knowledge and also the reference of singular terms in the scope of the propositional attitude of 'knowledge that'. The reference of such singular terms can be discussed in the course of discussing certain problems about identity and quantification in the logic of knowledge. So we turn to the logic of knowledge and the work of the most important philosopher in that field, Hintikka.¹

Hintikka's interest in modal logic, as we have seen, is consequent upon his belief that modal logic can be used to provide explanatory models for propositional attitudes. We saw in an earlier section² how the problems of designation and cross world identity arise in epistemic modalities. The particular modality with which we dealt was the modality of belief, the "doxastic" modality as Hintikka calls

¹J. Hintikka, Knowledge and Belief,

²Supra, p. 9.

it. Problems of exactly the same sort are to be found when dealing with the modality of knowledge, for which Hintikka reserves the term "epistemic".

In his many publications on the logic of knowledge Hintikka has always maintained that quantifying into epistemic contexts is of vital importance to his understanding of the epistemic propositional attitude. One of the things to be discussed in this chapter is the explication of quantifying into epistemic contexts. Hintikka's views will be used as the basis for that discussion. We need to discuss Hintikka's views about quantifying into epistemic contexts because they are pertinent to what Russell says about naming and being acquainted.

We have already seen how Russell maintained that a logical name could only be applied to a particular with which the speaker is acquainted. Hintikka has attempted to appropriate some of what Russell maintains for his epistemic logic.

The truth conditions of
 (8) $(\exists x) \underline{a} \text{ knows that } (\underline{x} = \underline{b})$
 with a contextual¹ quantifier are the same as the truth conditions of
 (9) $\underline{a} \text{ knows } \underline{b}$
 in ordinary language. ... The framework we have to rely on here is that created by one's first-hand knowledge of people and things, ... Another way of describing the force of (8) and (9) is of course to say that they are equivalent to
 (9)* $\underline{a} \text{ is acquainted with } \underline{b}$.²

The quantifier $(\exists x)$ is not to be confused with (Ex) .

¹The values of the variable for the 'contextual' quantifier are perceptual objects, i.e. objects which can be individuated in some demonstrative way.

²J. J. Hintikka "Objects of Knowledge and Belief: Acquaintances and Public Figures", *Journal of Philosophy* Vol. LXVII, 21, Nov. 1970, pp. 878-879.

The former has as its domain perceptual objects, that is objects which can be individuated in some demonstrative way. At any rate, that is what Hintikka says of $(\exists x)$. The other quantifier (Ex) was originally introduced as a simple free logic quantifier but modified as discussion progressed. Eventually it was seen as ranging over the individuals of whom it is known who or what they are.

At first Hintikka said that the formula

$(Ex) \ a \text{ knows that } (x = b)$

was the appropriate formalization of "a knows who b is." More recently he has been emphasising the importance of acquaintance to knowing who someone or something is. So he has introduced the quantifiers. In this way he is trying to appropriate something of Russell's view.

In order to show the relevance of Hintikka's efforts here for this essay we will need to set out briefly the epistemic logic which Hintikka began to construct in Knowledge and Belief, and we will concentrate our attention on those sections of the logic which deal with the question of what it is to know who or what somebody or something is.

Epistemic logic

Hintikka's epistemic logic seems to have no fixed form. Each time he writes about it he suggests changes. But there are some things which have remained fixed through all the debates, alterations and revisions. I shall set out an early simple form of his epistemic logic, which shall be called 'HS4'. The sort of notation which we have been using will be used, and where possible the rules we have set out above.

In HS4 two new improper symbols are introduced: \underline{K} , \underline{P} . These occur, like modal operators, in formulae of the form $\underline{K}_a \underline{A}$ and $\underline{P}_a \underline{A}$, where ' a ' is an individual constant, and ' \underline{A} ' is any formula. ' $\underline{K}_a \underline{A}$ ' is read as ' a knows that \underline{A} '. ' $\underline{P}_a \underline{A}$ ' is defined as ' $\sim \underline{K}_a \sim \underline{A}$ ', and is read as 'It is possible, for all that a knows, that \underline{A} '.

HS4 is set out in terms of model set model system semantics, and what we call 'satisfiability' Hintikka calls 'defensibility'. There is a set of consistency conditions. The propositional operator conditions are effectively as we have them: (C, \sim) , $(C, \sim \sim)$, (C, \supset) and $(C, \sim \supset)$. The rules for quantifiers are free logic rules. Only the quantifiers E and U are used. The rules (C, E_0) and (C, U_0) are adopted. These are:

- (C, E_0) If $(E\mathbf{x})\underline{A} \in \mu_n \in \Omega$ then $(E\mathbf{x})(\mathbf{x} = \underline{a}) \in \mu_n$ and $(\underline{A}(\underline{a}/\mathbf{x})) \in \mu_n$ for some individual constant \underline{a} .
- (C, U_0) If $(U\mathbf{x})\underline{A} \in \mu_n \in \Omega$ and if $(E\mathbf{x})(\mathbf{x} = \underline{a}) \in \mu_n$ then $(\underline{A}(\underline{a}/\mathbf{x})) \in \mu_n$.

We also set out again

$$(C, \text{self} \neq) \quad \underline{b} \neq \underline{b} \notin \mu_n$$

There is also a restricted version of $(C, =)$

$$(C, K=) \quad \text{If } \underline{A} \in \mu_n \text{ and } \underline{a} = \underline{b} \in \mu_n \text{ and } \underline{A} \text{ is an atomic formula or identity then } (\underline{A}(\underline{a}/\underline{b})) \in \mu_n$$

The restriction here ensures that no modal operators occur in \underline{A} ,

'modal operators' here referring to the epistemic operators \underline{K} and \underline{P} .

There are also the rules

$$(C, =K) \quad \text{If } \underline{K}_a \underline{A} \in \mu_n \text{ and } \underline{a} = \underline{b} \in \mu_n \text{ then } \underline{K}_b \underline{A} \in \mu_n$$

$$(C, =P_a) \quad \text{If } \underline{P}_a \underline{A} \in \mu_n \text{ and } \underline{a} = \underline{b} \in \mu_n \text{ then } \underline{P}_b \underline{A} \in \mu_n$$

The rules for epistemic operators are analogous to the rules we used for M^1Q = except that there is no analogue for $(C, \Box \Box *)$. The

systems we set out above have been analogous to S5, but HS4 is analogous to S4. The rules are

- (C.P*) If $\frac{P}{a}A \in \mu_n \in \Omega$ then there is in Ω at least one model set, such as μ_m , such that $A \in \mu_m$ and μ_m is an alternative to μ_n with respect to a in Ω .
- (C.K) If $\frac{K}{a}A \in \mu_n \in \Omega$ then $A \in \mu_n$.
- (C.KK*) If $\frac{K}{a}A \in \mu_m \in \Omega$ and μ_m is an alternative to μ_n with respect to a in Ω then $\frac{K}{a}A \in \mu_n$.

It can be seen that the alternativeness relation is made relative to the knower.

The general picture that this logic gives is that if a knows that A , then A is true and all alternative worlds must be consistent with a 's knowing that A . And if we want to hold that it's possible, for all that a knows, that A , then A will be true in at least one possible world. These worlds are related to one another in terms of what's possible for all that a knows. In this sense we can say that they are 'epistemic' alternatives. So, in discussing the propositional attitude of knowing that, we "restrict our attention to those possible worlds in accordance with this attitude."¹

One particular point, which we have mentioned already in discussion,² which should be borne in mind is that if a knows that b is F then b will be F in all epistemic alternatives and it must be the same person (or individual) who is called ' b ' in all these alternatives. Individual constants have to be rigid designators or we will be in danger of losing the individual of whom a knows something as we move from epistemic alternative to epistemic alternative.

¹"Semantics for Propositional Attitudes", op.cit. p. 25.

²Supra p.

At this point we digress slightly to point out that there are some major difficulties in HS4. There are two main problems associated with the purely propositional portion of HS4, and there is a cluster of problems in the quantificational part of the system. In the propositional portion of HS4 there is, first, the KK-thesis which makes it a feature of the logic that $\underline{K_a p} \supset \underline{K_a K_a p}$ is self-sustaining.¹ Secondly, there is that feature of the logic which makes it indefensible to say of any person a that he does not know T where T is any tautology of propositional calculus or any self-sustaining formula of HS4. $\sim \underline{K_a T}$ is indefensible, so $\underline{K_a T}$ is self-sustaining. This feature of the logic is often called the "deductive omniscience" feature.²

Elsewhere³ I have dealt with these two problems and have shown that although these problems show HS4 to be an unsatisfactory logic, a logic can be constructed which overcomes these problems and is far more appropriate as an explanatory model for understanding "the workings of our ordinary language."⁴ Hence, we will not be concerned with these problems here. But the reconstructed logic is set out in Appendix IV in both axiomatic and semantic forms. These problems are not central to our concerns here. Our main concern is for the quantificational part of HS4.

¹There has been considerable debate about this thesis: See especially: Synthese Vol. 21 (1970).

²J. Hintikka, "'Knowing that one knows' Reviewed", *Ibid.* p. 142.

³R.A. Girle, "Epistemic Logic, Language and Concepts", Logique et Analyse 63, 1972.

⁴J. Hintikka, "Epistemic Logic and the Methods of Philosophical Analysis", Australasian Journal of Philosophy 46 (1968), p. 40.

Quantifying into epistemic contexts

I have discussed the problems of the quantificational part of Hintikka's logic in the paper "Quantifying into Epistemic Contexts".¹ In that paper I show that a radical modification of Hintikka's dictionary, his recommendations for formalizing, will provide us with a simple and more straightforward epistemic logic than the complex, and even contorted, one he is still modifying. Indeed, the system set out above is not the system arrived at by the end of Knowledge and Belief. HS4 is only the starting point.

But the reasons for all the modifications are to be found in two main areas. First, there is lack of clarity about the domains of quantifiers. Secondly, there is Hintikka's quite idiosyncratic way of treating formulae in which a quantification outside the scope of an epistemic operator binds a variable within the scope.

The lack of clarity about the domains of quantifiers displays itself, at first, in a lack of clarity at every point that the quantifiers range only over existing individuals whilst the constants can designate non-existent individuals. To be precise, "existing" means "existing now". If quantifiers range over existing individuals then they range over individuals who exist at present. It might be argued that this is too restrictive, that we should allow the quantifiers to range over individuals which have, at some time, existed or will at some time exist. Instead of reading

$(\exists x)Fx$

as Something which exists now is F,

¹R. A. Girle, "Quantifying into epistemic contexts", Logique et Analyse 65-66, 1974, pp. 127-142. Some of the argument from that paper is reproduced here, of necessity, for the sake of continuity.

we should allow the reading

Something which has existed or exists or will exist is F .

It will certainly be easier for our purposes if we do allow the more generous reading of the quantifier. We can then allow that

$(\exists x)(x = \text{Socrates})$

and $(\exists x)(x = \text{Nixon})$.

In order to be precise we will use "exists" as an abbreviation for "has existed or exists or will exist". We will use an asterisk on the cognates of "exists" to make them 'timeless' also.

Our main point is not that the domain of quantification in free logic must be the set of individuals which exist right now, but that the domain must be some set of 'existent' individuals from the present, the past or the future, or even from a conjectured universe.

We will adopt the convention that the domain of quantification is of individuals which exist*. In the light of that we turn to some cases of reading formulae.

Consider the following less problematic cases:

(1) $\underline{K_a} (\underline{x}) Fx$

(2) $\underline{K_a} (\exists x) Fx$

(3) $\underline{K_a} (\exists x)(x = b)$

(i) \underline{a} knows that everything is \underline{F} .

(ii) \underline{a} knows that at least one thing is \underline{F} .

(iii) \underline{a} knows that \underline{b} exists*.

For the sake of precision we must say that only (3) and (iii) can be associated. To associate (1) with (i), and (2) with (ii) is not rigorous enough. It takes point away from having a free logic.

We need

(i') a knows that every existing* x is F , and

(ii') a knows that at least one existing* x is F ,

to associate with (1) and (2) respectively. Some awareness of this point is to be seen in Hintikka's article "'Knowing Oneself" and other problems in Epistemic Logic".¹

This lack of clarity about the domain of quantification has also been admitted more recently, and quite explicitly, when Hintikka wrote of what was said in Knowledge and Belief,

It was in effect said that in expressions like '(Ex) $\neg a$ knows that ($h = x$)' (i.e., in expressions in which one quantifies into a knowledge context) the bound variable in a sense ranges over individuals known to a . What was intended was not the set of a 's acquaintances, but something that can be expressed more appropriately² by speaking of individuals of whom a knows who they are.

But in either case the domain of the quantifier would be different in $(Ex)K_a Fx$ and $(Ex)K_b Fx$. This would make the interpretation of formulae like

$$(Ex)(Ey) \neg [K_{a-b} (x = y) \& (y = c)]$$

immensely difficult.

If we accept that the quantifier quantified over those individuals of whom we are to say that a knows who they are, instead of always quantifying over existing* individuals, then we should read (3) as:

(iii) a knows that there is at least one person out of those of whom a knows who they are who is b .

This is certainly not the same as a knows that b exists* (or exists), and the logic is no longer free in any sense.

¹In Theoria, Vol. 32 (1966) pp. 4-5.

²J.J. Hintikka: "Objects of Knowledge and Belief: Acquaintances and Public Figures." The Journal of Philosophy, 67, 21, 1970, p. 880.

This lack of clarity about the domain of quantification makes the problem of understanding quantification into epistemic contexts more difficult than it should be. We turn to that problem now, and look at formalizations. Consider the formulae and readings:

- (4) $(\underline{x})K_{\underline{a}} Fx$
- (5) $(\underline{Ex})K_{\underline{a}} Fx$
- (6) $(\underline{Ex})K_{\underline{a}}^3 (\underline{x} = \underline{b})$
- (iv) Everyone known to \underline{a} is known by \underline{a} to be \underline{F} .
- (v) Someone known to \underline{a} is known by \underline{a} to be \underline{F} .
- (vi) \underline{a} knows who \underline{b} is.

In Knowledge and Belief (4) (5) and (6) were associated with (iv) (v) and (vi) respectively. Subsequently Hintikka decided that it would be better to associate (iv) with

$$(4') (\underline{x})(\underline{x} = \underline{x} \supset \underline{K}_{\underline{a}} Fx).^1$$

All of these associations can be seen to be counter intuitive, especially when we try to build up the associations from the well formed parts. For example, we can easily build up the association of (1) with (i). $(\underline{x}) Fx$ associates with:

Every existing* \underline{x} is \underline{F} .

$\underline{K}_{\underline{a}} Fx$ becomes:

\underline{a} knows that every existing* \underline{x} is \underline{F} .

Let us begin therefore as follows:

- (7) $\underline{K}_{\underline{a}} Fb$
- (8) $\underline{K}_{\underline{a}} (\underline{b} = \underline{c})$
- (vii) \underline{a} knows that \underline{b} is \underline{F} .
- (viii) \underline{a} knows that \underline{b} is \underline{c} .

If we associate (7) with (vii), and (8) with (viii), we can then

¹Op.cit. p. 33 ff.

proceed to find out what happens with the introduction of quantifiers. On the basis of the association of (7) with (vii) we would read (5) as:

- (ix) There is at least one existing* x such that a knows that x is F .

(ix) does not say that what a knows is known to be F , but that what a knows to be F actually exists*. Another way of putting this would be

- (x) One of what a knows to be F , exists*.

For example, a may know that Romeo, Juliet and Caesar are characters in Shakespeare's plays, so we can say:

- (Ex) $K_a x$ is a character in Shakespeare's plays.

Similarly we would read (6) as:

- (xi) There is at least one existing* x such that a knows that x is b ,

or as (xii) What a knows as b , exists*.

(xi) does not say that a knows b exists* (iii), nor that a knows who b is (vi) (unless knowing who someone is is knowing their name), but that something which a calls ' b ' happens to exist*. A case in point would be someone who has heard of Hamlet but does not know whether or not he exists*.

Also we would read (4) as:

- (xiii) Each and every existing* x is such that a knows x is F .

For all practical purposes we could not truthfully substitute the name of some human being for a in (xiii). The name of some omniscient (in some sense) being would be required. We can note here that while we have read (1) $K_a (x) Fx$ as indicating that a knows a universal generalization, we have refrained in (4) $(x) K_a Fx$ from attributing to a any knowledge of the universal generalization $(x)Fx$. It just

happens that when we consider the list of things of which a knows that they are F we discover that they exhaust the domain of existing^a things.

Although we could accept the association of (4) with (xiii) (5) with (x), and (6) with (xii) for the sake of intuitive elegance, this will not be a sufficient reason for such an acceptance, even if it is a weighty reason. Reasons of a more conclusive nature can be found if we consider the questions of the domain and scope of the quantifiers and the scope of the epistemic operator.

Before going further we should note that one way of seeing Hintikka's suggestions about the domain of quantifiers is to see it as a suggestion that not only should the domain change from person to person, but also according to whether or not the quantifier binds a variable in the scope of an epistemic operator when the quantifier is outside that scope. The parentheses in the quotation above, "(i.e., in expressions in which one quantifiers into a knowledge context)", could be read this way.

So we would read (1), (2) and (3) as (i), (ii) and (iii). But (4) (5) and (6) would have to be treated quite differently. For example, we should read (6) $(\exists x)K_a(x = b)$ as:

(xiv) At least one person from those of whom a knows who they are a knows to be b.

(b) should be read in this way because (6) is of the form $(\exists x)K_a A$ where the quantifier ranges over the individuals of whom someone knows who they are. So $(\exists x)A$ should read as

(xiv') At least one person of those of whom someone, say a, knows who they are is such that A.

Hintikka indicates that if the formula is of the form $(\exists x)K_a A$, then the someone who knows who or what things are is a. We then modify

(xiv') to bring it closer to (6):

(xiv'') At least one person, \underline{x} , of those of whom \underline{a} knows who they are is such that $\underline{K}_{\underline{a}}(\underline{x} = \underline{b})$.

This then gives (xiv) as suggested.

But, if we read (6) as (xiv) then we should not read (6) as: \underline{a} knows who \underline{b} is. It might be the case that (xiv) is true, but it does not follow that \underline{a} knows who \underline{b} is.

For example, let us draw up a list of individuals such that \underline{a} knows of each who he is. We declare this list to be the domain of quantification. In that list there is the name ' \underline{c} ', which is read as 'Dr. Jekyll'. So \underline{a} knows who \underline{c} is. Now, let it be the case that \underline{a} has heard of Mr. Hyde, but does not know who he is. It can therefore be truly said of \underline{a} that

(9) $(\exists \underline{x}) \underline{K}_{\underline{a}}(\underline{x} = \underline{b})$

where ' \underline{b} ' is read as 'Mr. Hyde'. This formula is true of \underline{a} because, \underline{a} does not know it, \underline{b} does designate one of the individuals of whom we can say that \underline{a} knows who he is (Dr. Jekyll).

Since the quantification is outside the scope of the epistemic operator in (9), we cannot assume that \underline{a} does know the information conveyed by the quantification. We have been observing a general principle, as it were, that when the quantifier, no matter what its domain, is outside the scope of the epistemic operator it is best to read the bound variable in such a way that we make clear that, for example in (9), \underline{a} does not necessarily know that the abstraction predicate for the domain of the quantifier is predicated of the individual designated by \underline{x} . In the case of (9) read as (xiv) the abstraction predicate is " \underline{a} knows who ... is." In a standard free logic the abstraction predicate is "...exists", or in our case it is "...exists*".

Nevertheless, there could be an objection to our whole procedure above in deriving readings for formulae (4) (5) and (6) from the well formed formulae which feature in their construction. Although it is not clear, we could read the parentheses in the quotation above from "Objects of Knowledge and Belief: Acquaintances and Public Figures" as indicating that when a quantifier quantifies into an epistemic context then it should be treated in a fashion different to when it does not. In fact, Hintikka does just this. Not only does he change the domain of the quantifier from context to context, but he proposes special rules for certain formulae. Because of what Hintikka actually does Åqvist contends that he has infinitely many quantifiers in his logic.¹

There is a more telling passage in Knowledge and the Known where Hintikka claims that the truth value of statements of the form

$$(10) \quad (\exists x) \underline{a} \text{ knows that } (\underline{b} = \underline{x})$$

"is not determined by the truth-value of simpler statements involving no quantifiers. Our friend a's knowledge of any number of de facto identities of the form 'b = d' does not by itself determine whether he knows b or not."² And Hintikka claims that if a knows b then a knows who b is, and conversely.

So if we want to test the truth of (10) we need not bother to instantiate. All we will get are formulae of the form $\underline{K}_a(\underline{b} = \underline{d})$, and no number of these will help.

¹L. Åqvist, "Modal Logic with subjunctive conditionals and dispositional predicates", Journal of Philosophical Logic, Vol 2 (1973) p. 53. See also M. Rennie, op.cit. p. 115.

²J. Hintikka, Knowledge and the Known, Reidel Dordrecht, 1974, p. 222.

There are general difficulties about going further with the discussion of Hintikka's epistemic logic. One difficulty is that the logic is not in final form. Revisions are being made continuously, and new problems are being found. Until Hintikka can finalise his logic it will suffice for us to operate with a less problematic system --- one which does not have infinitely many quantifiers.

We return, therefore, to the original HS4 with the revised recommendations for formalising and interpreting. We assume that the quantifiers have as their domain the domain of existing* individuals, whether a knows who they are or not. We also assume that individual constants (not definite descriptions) are rigid designators and can be empty. There, therefore, seem to be no real problems with formulae (1)-(3).

We can give added strength to our proposed readings of (4) (5) and (6) by either of two methods. On the one hand we can take a sample of representative formulae, test them for self-sustainability, and see whether the readings we propose support the result of formal testing. On the other hand we can apply our theory of proper names in a discussion of the truth-conditions of (4) (5) and (6) and related formulae.

Taking the first method first, we consider the Barcan formulae and its converse for epistemic logic, the analogous formulae with existential quantifiers, the analogous formulae with existential quantifiers and identity, and the formulae which will enable us to decide on the validity of:

- (A) Premise $K_a (b = c)$
 Premise $(E_x) (x = b)$
 Conclusion $(E_x) K_a (x = c)$

and.

- (B) Premise $K_a (b = c)$
 Premise $(E_x)(x = b)$
 Conclusion $K_a (E_x)(x = c)$

It turns out that neither the Barcan formula nor its converse are self-sustaining in the system. Counter models are to be found in Appendix III. This accords with our readings and intuitive evaluation of the formulae. The Barcan formula is

$$(11) \quad (U_x)K_a Fx \supset . K_a (U_x)Fx.$$

This will be falsified if the antecedent is true and the consequent false. On our readings, the antecedent would read as "Everything which exists* is what a knows to be F." The consequent would be false if a did not know that everything existing* was F, that is, if he believed of one of the things which he knew to be F that it was non-existent*. This is certainly possible.

Consider then the formula:

$$(12) \quad K_a (U_x) Fx \supset . (U_x) K_a Fx.$$

The antecedent reads as "a knows that everything which exists* is F." The consequent will be false if there exists* something of which a does not know that it is F. This is possible, for unless a is omniscient we cannot guarantee the truth of the consequent.

When we turn to the analogues of (11) and (12) with existential quantifiers we find that they are not self-sustaining either. This accords with our intuitive evaluation of the implications. Consider first:

$$(13) \quad K_a (E_x) Fx \supset . (E_x)K_a Fx$$

This could be the formalization of a conditional, the antecedent being "a knows that there actually existed at least one person who was

a character in Shakespeare's plays," the consequent being "there actually existed at least one person of whom a knows that he was a character in Shakespeare's plays."

Clearly, someone could know that Shakespeare wrote plays in which historical characters appeared without knowing who any of the characters were in any play. Of such a person we could not say that, of the characters he knows to be Shakespearean, at least one existed, because he knows none of the Shakespearean characters. Of such a person we can affirm the antecedent of (13) and deny the consequent.

Similarly for

$$(14) \quad (E_x) K_a Fx \supset K_a (E_x) Fx$$

There is nothing counter-intuitive in saying, "The person known by a as a character in a play actually existed but a does not know that some characters in plays actually exist*." Once again we can affirm the antecedent and deny the consequent.

The formulae analogous to (11) and (12) with existential quantifiers and identity are

$$(15) \quad (E_x) K_a (x = b) \supset K_a (E_x)(x = b)$$

and its converse (16). In HS4 (15) is not self-sustaining, but its converse is. We have already explained that the antecedent of (15) can be true without a's knowing that b exists. But if a knows that b exists then it does follow that what a knows as b exists.

As for the arguments (A) and (B), on our reading (A) is valid but (B) is not. The system HS4 shows just this.

Oratio recta and Oratio obliqua

We now turn to the application of our theory of proper

names to the discussion of truth conditions for (4) (5) and (6).

First we consider

$$(5) \quad (Ex)K_a Fx.$$

Our claim is that (5) will be true if $K_a Fx$ and $(Ex)(x = X)$ are true where X is some individual constant, that is if a knows that X is F and X exists (whether a knows that or not). We would want to say further that $K_a Fx$ is true (where X is some individual constant) if, and only if, a knows that something, named by some name, is F . It is not sufficient that a knew that something is F , but what he should know is that Fx , where X is some name. Let us assume that the name is ' b ' (in typical existential instantiation fashion).

If a knows that b is F , then, with one proviso, on our account, a understands that ' b ' is a name, and hence that there is something named by ' b ', and whatever it is that is named by ' b ' is F . That one proviso is of tremendous importance, it is that we are considering what a knows in the sense of knowledge by description. We are considering what a would affirm as knowledge, in his terms, especially the singular terms.

This is not to deny that people can know people, things or places. Nor is it to deny that a person can know of some object that it has properties without ever giving that object a name or putting into words the knowledge of that object's properties. Sometimes accounts are given of what a person knows without taking account of the terms in which that person would enunciate what he knows. For example, we might say "Jones knows that the centrifuge is switched on by the door", and then qualify this by adding, "but he doesn't know that it a centrifuge". Perhaps Jones just saw someone switch it on one day. If Jones were asked to say what he knows about switching on

"that machine over there", he may well name the machine, or use the demonstrative. It is what he would say or affirm that concerns us.

When we formalize into formulae of the form $K_a A$ we could say that the ' $K_a \dots$ ' stands for ' a knows that the proposition expressed by \dots is true'. This is to emphasise the importance of the terms in which a would articulate his knowledge. But this raises problems for (5) unless we take the quantifier in a substitutionary fashion. It is not clear that

- (xv) There exists some x such that a knows that the proposition expressed by ' Fx ' is true.

even makes sense. But

- (xvi) There is at least one name, ' x ', of an existing individual such that a knows that the proposition expressed by ' Fx ' is true.

does make sense. Since I have argued above that it does not really matter whether we interpret quantifiers substitutionally or objectually, and since I have argued in this section that we must concentrate on the terms in which a knower expresses his knowledge, it follows that there is good sense in interpreting the quantifiers substitutionally in epistemic logic.

If we do not do so then prima face there would be considerable difficulty in interpreting (5). At first sight it looks as though the only way of taking account of the terms which the knower would use or affirm is to accept an interpretation which takes account of names and predicates. The substitutional interpretation of quantification is in terms of names. If we do not adopt the substitutional interpretation of quantifiers, at first sight, it looks as though we are not taking account of the names which the knower would use or acknowledge.

If we adopt an interpretation which ignores the terms in which the knower would express his knowledge then it would be virtually impossible to deal with the problems of opaque contexts in any logic. The problem which is most important is the problem of the substitutivity of identicals.

It is generally agreed that the following argument is invalid:

(C) Jones knows that Cicero addressed the Senate.

Cicero is Tully

Therefore, Jones knows that Tully addressed the Senate.

But the argument will be invalid only so long as we insist that what we claim Jones knows should be put in terms of what he would say or affirm. If we do not so insist, then we could claim that the argument is valid even though Jones does not know that Tully is Cicero. The same is equally true of the other propositional attitudes such as 'believes that', 'remembers that', 'hopes that', and so on.

Because it is usually claimed that argument (C) is invalid it is said that we cannot intersubstitute identicals in contexts of propositional attitude. These contexts, together with quotational contexts, are said to be opaque. It is notable that propositional attitudes contexts are grouped with quotational contexts. There is an analogous argument to (C) which is:

(D) Jones said, "Cicero addressed the Senate"

Cicero is Tully

Therefore, Jones said, "Tully addressed the Senate".

This is clearly invalid. Here we don't have to insist on framing premises and conclusion in just those terms which Jones would use, we are reporting what he said. It is just because we do report what

he said that argument (D) is plainly invalid.

The analogy between (C) and (D) can be made stronger, not only by means of the general consensus as to what counts as an opaque context, but also if we allow that one can readily move back and forth from oratio obliqua to oratio recta. There seem to be good reasons for holding that one can readily move back and forth from oratio obliqua to oratio recta, and if one can readily so move then it can be argued that we are not forced to adopt the substitutionary interpretation of quantifiers.

Geach argues that we can so move. In Logic Matters he writes,

To describe the words and attitudes of others we may use either the oratio obliqua construction, as in:
 (1) James believes that his wife fears that she has cancer.
 or the oratio recta construction:
 (2) James believes "My wife's fear is 'I have cancer.'"¹

Geach acknowledges that philosophers do not like the oratio recta form and argue against its use. But he says,

It is clear in advance that these arguments are sophistical; for forms like (2) are common in all vernaculars (cf. the King James Bible) and perfectly intelligible, and translating from oratio recta to oratio obliqua and back is an easy exercise for an intelligent schoolchild, difficult as it would be to formulate exact rules for this.²

If we agree with Geach then (C) can be translated into

(C') Jones knows "Cicero addressed the Senate"

Cicero is Tully

Therefore Jones knows "Tully addressed the Senate".

Unfortunately (C') does not sit as well as (2). But with the small addition, in the case of 'knows' only, of 'is true' we can get

¹P.T. Geach, Logic Matters, Blackwell, Oxford, 1972 p. 167.

²Ibid. p. 167.

(C'') Jones knows "Cicero addressed the Senate" is true

Cicero is Tully

Therefore Jones knows "Tully addressed the Senate" is true.

(C'') reads well and falls in with Geach's cases. One might therefore assume that Geach's version of (xvi) would be somewhat like:

(xvi'') There is at least one name, 'x', of an existing individual such that a knows 'F_x' is true.

I say 'somewhat like' because Geach's example (2) has 'my' in the quotational context referring back to 'James' which is not in the quotational context.

This, it seems to me, is a most important feature of Geach's (2). Here we have what can be seen as a form of quantification into opaque context. A variable within the context is bound by something outside. But the whole sense of Geach's (2) is quite neutral about whether the binding of the variable is to be interpreted substitutionally or objectually. 'My' can be seen as a space for the name 'James' (in the possessive as is 'My') or can be seen as referring to that to which 'James' refers because 'James' binds 'my'.

An oratio recta reading of (xvi) could therefore release us from any bondage to a substitutionary interpretation of quantification. Such a reading of (xvi) would be

(xvi'') There is at least one existing individual of which a knows 'it is F' is true.

care must be taken to see the 'it' here as being bound by 'which' and not as a demonstrative. The use of 'x' in (xvi) makes the binding clear, so we won't reject (xvi). The important thing is that, care taken, we can move backwards and forwards between (xvi) and (xvi').

The argument suggested earlier, that we need a substitutional interpretation of quantification in order to take account of the

names a knower would use or acknowledge, is shown to be unsound if we allow this movement from oratio obliqua to oratio recta. Furthermore, we have seen that in allowing the movement back and forth we cannot ignore, in oratio obliqua, the terms a knower would use or acknowledge.

It seems to me that Geach's appeal to the vernacular and to the ease with which one can move back and forth between oratio recta and oratio obliqua constitute an excellent basis for his claims. But he also sets out to argue against some arguments of philosophers who do not want to allow moves back and forth. He says that the underlying prejudice against moves back and forth

is expressible as follows: "(2) is only language about language, and this can touch only the physical expression of attitudes, not describe the attitudes themselves."¹

Geach then sets out to 'dent' this prejudice by arguing about the nature of propositional attitudes and propositions. Whatever the virtues of his arguments, and I have little sympathy for them, it is certain that he has pinpointed the prejudice. The prejudice is a prejudice for declaring as much as possible in ordinary language to be metalinguistic. This prejudice when applied to Geach's (2) sees (2) as a meta-meta-linguistic statement. It is absurd to see Geach's (2) in this way, whatever one believes about propositions.

Returning then to formula (5), we claim that (xvi) gives us a way of interpreting (5) which can also be used for (4) and (6). Consider

$$(4) \quad (U_x)K_{\underline{a}} Fx,$$

and the interpretation given by either

(xvii) Each name, 'x', of existing individuals is such that a knows that the proposition expressed by 'Fx' is true.

¹Ibid. p. 168.

or a Geachean

(xvii') Each existing individual is such that a knows 'it is F'
is true.

These accord with our straightforward interpretation of (4) as (xiii)
or as

(xiii') What a knows to be F constitutes the totality of
existing things.

Consider also

(6) $(\exists x) \underline{K}_a (x = \underline{b})$

and the interpretation given by either

(xviii) There is at least one name, 'x', of an existing individual
such that a knows that the proposition expressed by
'x = b' is true.

or by

(xviii') There is at least one existing individual such that a
knows 'it is b' is true.

If a understands that 'b' is a name then a will know that b = b.

If b also happens to exist then (6) will be true. (6) will be false
if b does not exist or a does not understand 'b' to be a name. If a
does understand that 'b' is a name then a will understand, on our
account, that something is called 'b'. If a understands that some-
thing is called 'b' and b exists then (6) would be true on the
interpretation given by (xviii). Once again, so long as we are care-
ful to take the 'it' in (xviii') as bound by 'one existing individual'
and not as a demonstrative, then (xviii') also supports the reading
we have given to (6).

Frege's Problem

According to the interpretations given to epistemic contexts we can say that the two identities which troubled Frege have quite distinct cognitive value. Consider

$$(17) \quad K_a (\underline{b} = \underline{b})$$

$$\text{and } (18) \quad K_a (\underline{b} = \underline{c})$$

The former is interpreted as:

(xix) \underline{a} knows that the propositions expressed by ' $\underline{b} = \underline{b}$ ' is true and the latter as:

(xx) \underline{a} knows that the proposition expressed by ' $\underline{b} = \underline{c}$ ' is true.

In HS4 (17) is self-sustaining because of the deductive omniscience feature of HS4. In the system I propose, which is set out in Appendix III, (17) is not self-sustaining. Whilst one might not want to support deductive omniscience, a feature of Hintikka's logics which troubles even him,¹ one might want to argue that (17) should be self-sustaining. There is a sense in which everyone knows that $\underline{b} = \underline{b}$. (Similarly for some other things like $(p \supset p)$.) But (17) can be self-sustaining only if \underline{a} takes ' \underline{b} ' to be a name. \underline{a} might not have ever heard the name before. If \underline{a} has heard the word ' \underline{b} ', and understands it to be a name, then (17) will follow. (17) follows because if \underline{a} understands ' \underline{b} ' to be a name, then \underline{a} will know that what is named by ' \underline{b} ' cannot but be identical with what ' \underline{b} ' names.

But from an understanding that ' \underline{b} ' is a name and that ' \underline{c} ' is a name (18) does not follow. For (18) to be true \underline{a} has to know that what ' \underline{b} ' names is also named by ' \underline{c} '.

Our theory of proper names allows us to maintain that the

¹J. Hintikka, "Knowing that one knows' Reviewed", *Synthese* 21 (1970) p. 142.

sense and reference of proper names are the same in direct and indirect speech. We do not have to resort to the Fregean notion that the reference of a proper name in indirect speech is its own sense.

Conclusion

I have argued for a theory of proper names, and have argued that the theory is of help in dealing with certain philosophical problems. The theory is essentially a simple theory that emphasises the naming relation. Read in one way, the theory advocated is that names have no meaning. Names simply refer and their sense is that they refer. But if the role of names in statements is to be relevant to the truth conditions of statements, and it is, then we can say that names have meaning in context, and our theory shows how names play out their role.

The sense of a name is its relation to that to which it has been given. It was Mill's view that names had denotation but no connotation. In saying that names have sense I do not mean to say that names have connotation. If meaning is seen principally in terms of connotation then our theory implies that names have no meaning. As Wiggins says in a footnote to "Essentialism, Continuity and Identity",

Champions of the designational view of proper names defended here are sometimes asked, when (unlike Kripke) they present their view in an in other respects Fregean framework, what is the residual utility of the notion of sense? The answer is that the thing designated is still

in a different category from the sense of its name, and that sense is still needed to state the relationship of the meaning of the name and the meanings of the sentences in which it figures. Sense mediates between the truth grounds of the latter and the contribution of the former to truth-grounds.¹

The giving of a name is quite crucial to this theory.

There may be a stock of proper-names or individual constants in a language, but these are merely words or terms until they have been given to something. This is not just a matter of pragmatics either, a matter divorced from semantics. In standard set theoretic semantics one gives names to individuals by means of valuation functions. In substitutional semantics one assumes that the names are the names of the things of which we speak. Even in deduction, especially with Existential Instantiation, when one proceeds for example from

$(\exists x)(\forall y) Rxy$

to $(\forall y) Ray$,

what has been done "may be expressed in words saying 'consider by way of example one individual, let us name it a, of the kind which is asserted to exist by $(\exists x)(\forall y)Rxy$.'"²

Once a name has been given then it is fixed as one of the arguments of the naming function, and its sense is that it names just one individual.

On the basis of this theory we have considered the problems involved in cross-world identity. It has been shown that if we adhere closely to the theory then we can construct quantified modal logics, and deal with problems about identity in modal systems. We have seen especially, that in a variety of modal logics, if we adhere to the theory of names set out, then there is uniform agreement about the

¹Op.cit. p. 357, fn. 29.

²J. Hintikka "Quantification and the Picture Theory of Language", *The Monist* 53 (1969), p. 222 (my italics).

validity or self-sustainability of the formula

$$(a) \quad (U_x)(U_y)(x = y \supset \Box(x = y)).$$

This formula remains self-sustaining no matter how the domains of individuals are related from world to world. In this way our theory supports the notion that a name is a rigid designator.

The theory advocated has several advantages over other theories. Its advantages over Frege's theory are that first, the sense of a name is determined and not dependent on differing users, and second, that the referent of a name is the same in all contexts. The advantages over Russell's theory are that first, the term 'name' itself is not so narrowly defined, and second, that non-existent entities can be named.

Above all, the theory accords with ordinary language and provides a basis for formalising into first order, modal and epistemic logics.

APPENDIX I

A type theory basis for a theory of proper names

In order to develop the theory that the sense of a proper name is the relation established by the giving of that name we will make use of the simple theory of types. We begin by setting out Rennie's expanded and modified account¹ of Church's formulation of the simple theory of types². Rennie's account will then be expanded in certain respects.

We begin with an informal account of the syntax and semantics. There are three type symbols in Rennie's system, \circ , ι and κ : truth-values have type \circ , individuals have type ι , and possible worlds have type κ . Type symbols are recursively specified by:

1. \circ, ι and κ are type symbols
2. If α and β are type symbols, so is $(\alpha\beta)$.

The important feature of this notation is that functions from items of type β to items of type α have type $(\alpha\beta)$. For example, $(\circ\circ)$ is the type of one kind of truth function, and $(\circ\iota)$ is the type of first-order monadic predicates. Rennie argues that $(\circ\kappa)$ is the type of propositions³. Type symbols always appear as subscripts to variables and constants in formulae. For any type symbol α there are the following denumerably many variables:

$$f_{\alpha}, g_{\alpha}, h_{\alpha}, m_{\alpha}, n_{\alpha}, x_{\alpha}, y_{\alpha}, z_{\alpha}, f_{\alpha}^{(1)}, g_{\alpha}^{(1)}, \dots$$

There are also the two constants of fixed type:

$$N_{(\circ\circ)}, A_{((\circ\circ)\circ)}$$

For any type symbol α we have the constants:

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1. M.K.Rennie Some Uses of Type Theory in the Analysis of Language
Research School of Social Sciences, Canberra, 1974
 2. A.Church "A Formulation of the Simple Theory of Types" The Journal of Symbolic Logic (5) 1940 pp. 56-68
 3. Op.cit. pp 74-75

$$\Pi_{(o(o\alpha))}, \quad \iota_{(\alpha(o\alpha))}$$

There are also the improper symbols:

$$\lambda, (,) .$$

There are no other primitive symbols in the notation.

Every formula has an associated type, and formulae (wffs) and their types are recursively defined as follows:

1. Any variable v_α is a wff, and has type α .
2. Any constant is a wff, and has the type of its subscript.
3. If $A_{(\alpha\beta)}$ is a wff of type $(\alpha\beta)$ and B_β is a wff of type β , then $(A_{(\alpha\beta)}B_\beta)$ is a wff of type α .
4. If A_α is a wff of type α and v_β is a variable of type β , then $(\lambda v_\beta A_\alpha)$ is a wff of type $(\alpha\beta)$.

As an example of the way in which these rules work, consider truth-functional logic of the classical kind where only o is the basic type symbol. The variables take as their values only truth-values, and the only constants are $N_{(oo)}$ and $A_{((oo)o)}$. We let $D_o = \{1, 0\}$ for the domain of truth-values. Formation rule 1 matches the usual clause that every truth-functional variable (or "propositional variable" as they are usually called) standing alone is a wff. The constant $N_{(oo)}$ denotes that function from D_o to D_o such that if the argument is 1 the value is 0, and if the argument is 0 the value is 1, i.e. truth-functional negation. By rule 2 above $N_{(oo)}$ standing alone is a wff. This is not as in classical truth-functional logic. But $(N_{(oo)}x_o)$ is a wff by rule 3. $(N_{(oo)}x_o)$ is of type o and $(N_{(oo)}A_o)$ is also of type o if A is a wff of type o . This matches the usual clause in classical truth-functional logic that if A is a wff then so is $\sim A$, and so we abbreviate $(N_{(oo)}A_o)$ as $(\sim A)$.

Similarly we abbreviate

$$((A_{((\circ\circ)\circ)}^{B_{\circ}})C_{\circ}) \quad \text{as} \quad (B_{\circ} \vee C_{\circ})$$

since $A_{((\circ\circ)\circ)}$ is the inclusive disjunction operator of classical truth-functional logic. The reason we can use monadic functions to define disjunction, a dyadic function, is that

$$A^{(BXC)} \quad \text{and} \quad (A^B)^C$$

are set theoretically equivalent.

It should also be noted that in this notation the argument of a function is indicated in the typal subscripts by the right-hand type symbol and the value by the left-hand type symbol. In any pair, $(\alpha\beta)$, α indicates the values and β the arguments. For this reason we have adopted the convention, in any set-theoretic notation, of letting the left-hand members of ordered pairs, in sets of ordered pairs which represent functions, be the values of functions.

As well as the abbreviated forms $(\sim A_{\circ})$ and $(B_{\circ} \vee C_{\circ})$ we also use $(B_{\circ} \& C_{\circ})$, $(B_{\circ} \supset C_{\circ})$ and $(B_{\circ} \equiv C_{\circ})$ in the usual way.

When we turn to first-order quantification theory we have variables of type ι as individual variables taking their values from a non-empty domain D_{ι} . Variables of types $(\circ\iota)$, $((\circ\iota)\iota)$, $(((\circ\iota)\iota)\iota)$, ... serve as variables for monadic, 2-adic, 3-adic, ... predicates respectively. In order to introduce quantification we first introduce the improper symbol λ .

Informally, λ is an operator of functional abstraction, and $(\lambda v_{\beta} A_{\alpha})$ denotes a function. The function consists of a set of ordered pairs, one pair for each value that v can take in D_{β} . For example:

$$\lambda x(x^2) = \{ \langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 4, 2 \rangle, \langle 9, 3 \rangle, \langle 16, 4 \rangle, \dots \}$$

where x can take values in $\{0,1,2,3,4, \dots\}$ and x 's values provide the arguments of the function. The values of the function are provided by x^2 in the example. So

$$(\lambda x_i f_{(o_i)} x_i)$$

is a functional representation of a monadic predicate, such as 'R...', with a free variable, such as y , forming the expression

$$R_y \quad .$$

If ' R_y ' were taken as ' y is red', and y could take as its values the members of a set of billiard balls: $\{\text{plain, spot, red}\}$ then we could represent ' R_y ' as:

$$\{\langle 0, \text{plain} \rangle, \langle 0, \text{spot} \rangle, \langle 1, \text{red} \rangle\}$$

Clearly ' R_y ' is represented by a function of type (o_i) , a function from individuals to truth-values. Since the type of $(\lambda x_i f_{(o_i)} x_i)$ is (o_i) , it follows that we can have

$$\Pi_{(o(o_i))} (\lambda x_i f_{(o_i)} x_i)$$

or

$$\Pi_{(o(o_i))} (\lambda x_i A_o) \quad \text{which has the type } o \quad .$$

$\Pi_{(o(o_i))} (\lambda x_i A_o)$ has value 1 iff $(\lambda x_i A_o)$ has value 1 for all the arguments from D_i . So in the billiard ball example, since the values for R_y are not 1 in every case

$$\Pi_{(o(o_i))} (\lambda y_i R_{(o_i)} y_i)$$

would have the value 0. In other words, it is false that all the billiard balls are red. We abbreviate

$$\Pi_{(o(o_i))} (\lambda x_i A_o) \quad \text{as} \quad (x_i) A_o$$

$$\text{and } (\sim(x_i)(\sim A_o)) \quad \text{as} \quad (\exists x_i) A_o$$

The constants $\iota_{(\alpha(o\alpha))}$ are best explained by considering $\iota_{(\iota(o_i))}$. $\iota_{(\iota(o_i))}$ denotes a function whose value for a given subset of D_i as argument is a member of that subset unless the subset is null. If the subset is null then the value is an arbitrary

member of D_t . We abbreviate $\mathcal{L}_{(\iota(\mathcal{O}\iota))}(\lambda x_t A_o)$ as $(\exists x_t)A_o$. If the value of $\mathcal{L}_{(\iota(\mathcal{O}\iota))}$ is a unit set then it acts as a definite description operator.

Identity for any type is defined in two steps. First we abbreviate

$$(\lambda x_\alpha (\lambda y_\alpha (f_{(\mathcal{O}\alpha)}((f_{(\mathcal{O}\alpha)}x_\alpha) \supset (f_{(\mathcal{O}\alpha)}y_\alpha)))) \text{ as } Q_{((\mathcal{O}\alpha)\alpha)},$$

and then abbreviate

$$((Q_{((\mathcal{O}\alpha)\alpha)}A_\alpha)B_\alpha) \text{ as } (A_\alpha = B_\alpha)$$

In general we will assume that D_α is the set of items of type α , and $D_{(\alpha\beta)} = D_\alpha^{D_\beta}$. We adopt the abbreviations set out by Rennie, especially within type symbols. The most important of these is that parentheses will be omitted from type symbols according to a principle of association to the left. So $x_{\mathcal{O}\iota\iota}$ is an abbreviation of $x_{(((\mathcal{O}\iota)\iota)\iota)}$.

Church's formulation of the simple theory of types contains no individual constants or anything analogous. Here, as elsewhere, Church distinguishes between free and bound occurrences of variables, and, when the semantics are set out, the free occurrences of variables are treated as though they were occurrences of constants.

A type for symbols

Since we wish to give an account of proper names in terms of the naming function which relates a name to that to which the name is given, we need to be able to refer to the names themselves. Although names, and all the other symbols of a formal or natural language, are "part of the world"¹, and therefore members of D_t , it seems best to treat them as a separate category. We therefore introduce the domain of symbols: D_σ . It would seem then that all

1. M.J.Cresswell Logics and Languages, Methuen, London(1973)

we need to do is to add σ to the type symbols and we will then be able to use expressions of type σ to refer to the symbols of the formal language we have set out above. Unfortunately, it is not as simple as that. We cannot simply use σ as a surrogate for quotation marks. For example, to refer to the name of x_ι we use ' x_ι '. If, on the one hand, we simply replace the inverted commas with an additional subscript, σ , to get $x_{\iota\sigma}$, then we have a symbol which stands for something of the type of a function from the domain of symbols to the domain of individuals but not for a member of D_σ . If, on the other hand, we replace both the inverted commas and the subscript, in ' x_ι ' for example, with a σ subscript, to get x_σ for example, then it is not clear that x_σ stands for the name of (the symbol) x_ι or x_o or x_κ . Indeed, it looks as though x_σ could designate part of a primitive symbol, that is, x_σ could be seen as designating the x in all the variables of the form x_α . But this still leaves us with the problem of designating the names of x_ι , x_o and x_κ .

It is therefore best to proceed at first in a way which can be exemplified by saying that:

$$'x_\iota' = x_\nu$$

$$'x_o' = x_\tau$$

$$'x_\kappa' = x_\mu$$

We will concern ourselves at first with naming only those primitive symbols of our notation with the typal subscripts ι , o and κ . Syntactically, this is easily coped with by the simple process of adding ν , τ and μ to the type symbols. The recursive specification of the type symbols then becomes:

1'. $o, \iota, \kappa, \nu, \tau$ and μ are type symbols.

2'. If α and β are both type symbols, so is $(\alpha\beta)$.

Informally, the semantics would then be constructed to say that, for example:

$$'f_{\alpha\iota}' = f_{\tau\nu}$$

$$'II_{(\alpha(\alpha\iota))}' = II_{(\tau(\tau\nu))}$$

$$'x_{(\alpha(\kappa\iota)(\alpha\iota))}' = x_{(\tau(\mu\nu)(\tau\nu))}$$

The status of formulae such as $f_{\iota\nu}$ would then be quite clear, this would be a variable of the type of functions from the names of individuals to individuals.

What has been sketched out above could also be seen as the short (and expedient) way of dealing with part of a general programme which should be set out, but we are hardly likely to have to resort to the general programme. In a general system we could use the σ symbol if we used it to subscript the whole symbol. For example, the name of x_{ι} would be designated by $x_{\iota_{\sigma}}$. In other words, x_{ν} is to be seen as an abbreviation of $x_{\iota_{\sigma}}$, and x_{τ} is to be seen as an abbreviation of $x_{\alpha_{\sigma}}$, and x_{μ} is to be seen as an abbreviation of $x_{\kappa_{\sigma}}$. Similarly, using ' \rightarrow ' for 'is an abbreviation for', we have:

$$f_{\tau\nu} \rightarrow f_{\alpha\iota_{\sigma}}$$

$$II_{(\tau(\tau\nu))} \rightarrow II_{(\alpha(\alpha\iota))_{\sigma}}$$

$$x_{(\tau(\mu\nu)(\tau\nu))} \rightarrow x_{(\alpha(\kappa\iota)(\alpha\iota))_{\sigma}}$$

In each of these cases we must treat the symbol σ as subscripting not only the whole symbol, such as x_{ι} , but also the subscript. So the subscripted subscript, such as ι_{σ} , can be seen as what is abbreviated. The symbol $f_{\iota\nu}$ is not an abbreviation for $f_{\iota_{\sigma}\nu}$, but for $f_{\iota_{[\sigma]}\nu}$, the square brackets showing the real sense of $f_{\iota\nu}$. Therefore, in a generalized notation using σ we would also need to use square brackets. In the generalized notation we have the possibility of formulae such as $x_{\iota_{[\sigma]}}$ for ' x_{ι} ', and so on.

In order to prevent σ appearing as other than a subscript to subscripts the recursive specification of type symbols would have to read:

1". o, ι and κ are type symbols.

2". If α and β are type symbols, so is (α/β) .

3". If α is a type symbol, so is $[\alpha_\sigma]$.

Abbreviations are:

$$\nu \rightarrow [\iota_\sigma]$$

$$\mu \rightarrow [\kappa_\sigma]$$

$$\tau \rightarrow [o_\sigma]$$

If α contains occurrences of only o, ι and κ , then if $\alpha(X/Y)$ is the result of substituting X for Y in α at every occurrence of Y , then

$$\alpha(\nu/\iota)(\mu/\kappa)(\tau/o) \rightarrow [\alpha_\sigma]$$

It must be noted that this notation does not permit our referring, by means of it, to the parts of primitive symbols.

Naming and referring

It is possible with this notation to define a naming function as:

$$\lambda x_\iota(x_\nu) \rightarrow \lambda x_\iota(x_{\iota_\sigma})$$

For any individual, x_ι , this function will give us its name. But in order to prevent any one name from being the name of more than one individual, and so avoid some of the hazards of ordinary language, we need to define a function which will ensure that a given name names only one individual. The function is:

$$\lambda z_\nu(\gamma y_\iota)(z_\nu = (\lambda x_\iota(x_\nu))y_\iota)$$

The first definition is of "The name of ...", the second is of "The referent of ...". It is this second function which gives the 'meaning' or 'sense' of any proper name. The appropriate

predicate constructed out of this second function will be a dyadic predicate of type $((o_i)\nu)$:

$$\lambda z_\nu \lambda w_i ((\gamma y_i)(z_\nu = (\lambda x_i(x_\nu))y_i) = w_i)$$

This can be abbreviated loosely as:

w is named by z

or more strictly as:

$$\text{Nam}_{((o_i)\nu)}$$

We can also take the examples "The name of a " and "The referent of a " , and construe these as follows:

$$\begin{aligned} \text{(a) The name of a} &= [\lambda x_i(x_{i_\sigma})](a_i) \\ &= a_{i_\sigma} \\ &= 'a_i' \end{aligned}$$

$$\begin{aligned} \text{(b) The referent of a} &= (\gamma y_i)(a_\nu = (\lambda x_i(x_\nu))y_i) \\ &= \text{the individual named by } a_\nu \end{aligned}$$

Example (b) is the more important of the two in the formulation of a theory of proper names. By recognizing a word to be a proper name that word is understood to name some one individual, and it names that one individual simply in the sense that the name has been given to that individual. That a name stands in such a relationship is what gives a name its sense. The property of having one and only one referent could be said to be what it is for a name to have a sense. Seen in the right way the function

$$\lambda w_\nu (\gamma z_{i_\nu})(z_{i_\nu} w_\nu \quad (\gamma y_i)(w_\nu = (\lambda x_i(x_\nu))y_i))$$

can be seen as the function which gives the sense of any one name.

Identity and sense

In the light of this account of the sense of a proper name we can account for the difference between "a = a" and "a = b"

which so concerned Frege. His question was directed to discovering how a person's understanding of " $a=a$ " differs from " $a=b$ " in such a way that the truth of one is determined in a way specifically different to the way used for the other. And it is not necessary to provide, besides a theory of proper names, a theory of predicates such as '='. It makes perfectly good sense to say that the difference in understanding can be made clear in terms of $Q_{((o_i)_i)}$ and the sense of a_y and the sense of b_y . Whereas we understand " $a=a$ " to be logically true because it is equivalent to

$$((Q_{((o_i)_i)}(\gamma y_i)(a_y = ((\lambda x_i)(x_y))y_i)((\gamma y_i)(a_y = ((\lambda x_i)(x_y))y_i)))$$

but " $a=b$ " is equivalent to

$$((Q_{((o_i)_i)}(\gamma y_i)(a_y = ((\lambda x_i)(x_y))y_i)((\gamma y_i)(b_y = ((\lambda x_i)(x_y))y_i)))$$

which is not logically true. In order to discover whether the second is true or not, as a matter of fact, one would have to discover what individual a_y is the name of, and of what individual b_y is the name.

Divided reference

We have seen that a property is represented in this notation as a function from D_i to D_o . In this case by "property" we mean a monadic property. If we let

$$\lambda x_i R_{o_i} x_i$$

denote the function which will determine for any individual whether it is red or not then R_{o_i} is the property of being red and $R_{[o_i]}$ is the general term designating the property.

Two important questions to ask are, does $R_{\tau_y}(R_{[o_i]})$ refer to anything? and, in what sense might that reference be a divided reference? These questions are asked in order to clarify, if possible, Quine's notion of divided reference. Although we might

say that $R_{\tau\nu}$ refers to the function $R_{\alpha\epsilon}$, this is clearly not what Quine intends by his talk of the divided reference of general terms. What he means is that $R_{\tau\nu}$ refers to those members of D_{ϵ} such that, as arguments for the function $\lambda x_{\epsilon} R_{\alpha\epsilon} x_{\epsilon}$, the value true will be designated. The divided reference of $R_{\tau\nu}$ will then be members of some subset of D_{ϵ} .

The property of a general term of having a divided reference is of type $(\phi(\iota(\tau\nu)))$. We can therefore define

$$DRef_{(\phi(\iota(\tau\nu)))} =_{df} \lambda x_{\iota(\tau\nu)} (z_{\tau\nu}) (\exists y_{\epsilon}) (x_{\iota(\tau\nu)} z_{\tau\nu} = y_{\epsilon})$$

For the sake of this discussion we can be Aristotelian and assume that all general terms are predicable of at least one individual. The function indicates that a term has divided reference only if

$$(z_{\tau\nu}) (\exists y_{\epsilon}) (x_{\iota(\tau\nu)} z_{\tau\nu} = y_{\epsilon})$$

is true for that term. But we have now returned to a quantified formula. We must consider the semantics for quantification, and the crucial quantification is $(\exists y_{\epsilon})$.

APPENDIX II

The systems $\mu^i Q = (1 \leq i \leq 4)$

Primitive symbols:-

- (a) improper symbols: $\supset \sim \cup ()$
- propositional variables: $p_1, q_1, r_1, p_2, q_2, r_2, p_3, \dots$
- individual variables: $x_1, y_1, z_1, x_2, y_2, z_2, x_3, \dots$
- individual constants: $a_1^n, b_1^n, c_1^n, a_2^n, b_2^n, c_2^n, a_3^n, \dots \quad (n \geq 1)$
- m-ary predicate variables: $F_1^m, G_1^m, H_1^m, F_2^m, G_2^m, H_2^m, F_3^m, \dots \quad (m \geq 1)$
- predicate constant: $=$
- (b) individual constants: $a_1^0, b_1^0, c_1^0, a_2^0, b_2^0, c_2^0, a_3^0, \dots$

Formation rules:-

1. A propositional variable standing alone is a wff.
2. If F^m is an m-ary predicate variable, and if a_1^n, \dots, a_m^n are m individual constants (not necessarily distinct) then

$$F^m a_1^n a_2^n \dots a_m^n$$
 is a wff.
- 2'. If F^m is an m-ary predicate variable, and if a_1, \dots, a_m are m individual constants (not necessarily distinct) then

$$F^m a_1 a_2 \dots a_m$$
 is a wff.
3. If \underline{a} and \underline{b} are individual constants (not necessarily distinct) then

$$\underline{a} = \underline{b}$$
 is a wff.
4. If \underline{A} is a wff so is $\sim \underline{A}$.
5. If \underline{A} and \underline{B} are wffs and all the individual constants

occurring in A and B outside of atomic parts which are identities have the same superscript, then so is $(\underline{A} \supset \underline{B})$.

6. If A is a wff and x is any individual variable, then, provided that all the individual constants in A have the same superscript,

$$(\underline{Ux})(\underline{A}(\underline{x//a}))$$

is a wff, where $(\underline{A}(\underline{x//a}))$ is the result of substituting x for a at zero or more occurrences of a in A.

6'. If \underline{A} is a wff and \underline{x} is any individual variable then

$$(U\underline{x})(\underline{A}(\underline{x}/\underline{a}))$$

is a wff, where $(\underline{A}(\underline{x}/\underline{a}))$ is the result of substituting \underline{x} for \underline{a} at zero or more occurrences of \underline{a} in \underline{A} .

T. Terminal clause.

Formulae constructed according to rules 1. and 2. are called "atomic wffs", and formulae constructed according to rule 3. are called "identities". The atomic parts of any formula are the smallest well formed parts.

The normal definitions of $\&$, \vee , \equiv and E are used.

Also $(\underline{A}(\underline{X}/\underline{Y}))$ is the result of substituting \underline{X} for \underline{Y} at every occurrence of \underline{Y} in \underline{A} .

Model System:-

A Model System, Ω , is such that

$$\Omega = \{\mu_1, \mu_2, \dots, \mu_n, \dots\} \quad (n \geq 1)$$

and the members of Ω are model sets, i.e. sets of formulae. The conditions which determine the membership of a model set vary from system to system, and are drawn from:

(C. μ_n) If \underline{A} contains any individual constant \underline{a}^i occurring outside of an identity and $i \neq n$, then $\underline{A} \notin \mu_n$.

(C.DD) If \underline{a}_j^i and \underline{b}_m^k are individual constants, $(i \geq 1)(k \geq 1)$, in any member of a member of Ω and $i \neq k$ then for every μ_n in Ω $\underline{a}_j^i \neq \underline{b}_m^k \in \mu_n$.

(C.DD⁰) If \underline{a}_j^i and \underline{b}_m^k are individual constants, $(i \geq 0)(k \geq 0)$, in any member of a member of Ω and $i \neq k$ then for every μ_n in Ω $\underline{a}_j^i \neq \underline{b}_m^k \in \mu_n$.

(C. \sim) Not both $\underline{A} \in \mu_n$ and $\sim \underline{A} \in \mu_n$.

(C. $\sim\sim$) If $\sim\sim \underline{A} \in \mu_n$ then $\underline{A} \in \mu_n$.

(C. \supset) If $(\underline{A} \supset \underline{B}) \in \mu_n$ then either $\sim \underline{A} \in \mu_n$ or $\underline{B} \in \mu_n$ or both.

- (C. $\sim \supset$) If $\sim(\underline{A} \supset \underline{B}) \in \mu_n$ then $\underline{A} \in \mu_n$ and $\sim \underline{B} \in \mu_n$.
- (C.E_n) If $(\underline{Ex})\underline{A} \in \mu_n$ then $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$ for at least one individual constant \underline{a}^n .
- (C.E) If $(\underline{Ex})\underline{A} \in \mu_n$ then $(\underline{A}(\underline{a}/\underline{x})) \in \mu_n$ for at least one individual constant \underline{a} .
- (C.EV⁰) If $(\underline{Ex})\underline{A} \in \mu_n$ then $(\underline{A}(\underline{a}^0/\underline{x})) \in \mu_n$ or $(\underline{A}(\underline{b}^n/\underline{x})) \in \mu_n$ for at least one individual constant \underline{a}^0 or one individual constant \underline{b}^n .
- (C.U_n) If $(\underline{Ux})\underline{A} \in \mu_n$ then if \underline{b}^n occurs in any formula in μ_n then $(\underline{A}(\underline{b}^n/\underline{x})) \in \mu_n$.
- (C.U) If $(\underline{Ux})\underline{A} \in \mu_n$ then if \underline{b} occurs in any formula in any μ_m then $(\underline{A}(\underline{b}/\underline{x})) \in \mu_n$.
- (C.U⁰) If $(\underline{Ux})\underline{A} \in \mu_n$ then if \underline{b}^0 occurs in any formula in any μ_m then $(\underline{A}(\underline{b}^0/\underline{x})) \in \mu_n$.
- (C.self \neq) $\underline{b} \neq \underline{b} \notin \mu_n$.
- (C. $\stackrel{n}{=}$) If $\underline{A} \in \mu_n$ and $(\underline{a}_1^n = \underline{b}_j^n) \in \mu_n$ then $(\underline{A}(\underline{a}_1^n/\underline{b}_j^n)) \in \mu_n$.
- (C. $=$) If $\underline{A} \in \mu_n$ and all the atomic parts of \underline{A} are in the form of identities and $\underline{a} = \underline{b} \in \mu_n$ then $(\underline{A}(\underline{a}/\underline{b})) \in \mu_n$.
- (C. μ =) If $\underline{A} \in \mu_n$ and all the atomic parts of \underline{A} are in the form of identities, then for each individual constant, \underline{a}_j^i , which occurs in \underline{A} with $i \neq n$, $\underline{A} \in \mu_i$, and if \underline{a}_j^i occurs in any formula in μ_m then $\underline{A} \in \mu_m$.
- (C. $=$) If $\underline{A} \in \mu_n$ and $\underline{a} = \underline{b} \in \mu_n$ then $(\underline{A}(\underline{a}/\underline{b})) \in \mu_n$.

In general we let 'S(Ω)' stand for ' Ω is a model system whose membership is determined by a set of rules for system S', or for ' Ω is an S model system.' Furthermore, if S(Ω) then the membership of Ω will be determined by a set of conditions C^S. We will denote C ^{$\mu^i Q$} by 'Cⁱ', ($1 \leq i \leq 4$).

A set of formulae, λ , is said to be satisfiable (consistent) in a system S iff the set is included in some μ_n in some S.

model system. We let 'SSat (λ)' stand for ' λ is satisfiable in S'. So we have

$$\text{SSat}(\lambda) \equiv (\exists \Omega)(\exists n)(S(\Omega) \& \mu_n \in \Omega \& \lambda \subseteq \mu_n)$$

If λ is satisfiable in S then we say that each member of λ is satisfiable in S, or just 'satisfiable' if the context makes clear which S we intend.

A formula is said to be logically true in S or self-sustaining in S iff the unit set of its negation is not satisfiable in S and the formula is satisfiable in S. When the context makes it clear which system is under consideration we will simply say that a formula is self-sustaining. We let 'SSelf-sus(\underline{A})' stand for ' \underline{A} is self-sustaining in S'. So we have

$$\text{SSelf-sus}(\underline{A}) \equiv \text{SSat}(\{\underline{A}\}) \& \sim(\exists \Omega)(\exists n)(S(\Omega) \& \mu_n \in \Omega \& \{\sim \underline{A}\} \subseteq \mu_n)$$

A formula will therefore be not self-sustaining in S iff either the formula is not satisfiable in S or its negation is satisfiable in S.

The primitive symbols for $\mu^1 Q =$ when i is either 1,3 or 4 are in section (a) of the primitive symbols as set out on page 193 of this Appendix.

The primitive symbols for $\mu^2 Q =$ are in sections (a) and (b) of the primitive symbols as set out on page 193 of this Appendix.

The formation rules for

$\mu^1 Q =$ are 1, 2', 3, 4, 5, 6' and T (see p 193)

$\mu^2 Q =$ are 1, 2, 3, 4, 5, 6 and T' (see p 193)

$\mu^3 Q =$ are as for $\mu^2 Q =$

$\mu^4 Q =$ are as for $\mu^2 Q =$

$$C^1 = \{(C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset), (C.E), (C.U), (C.\text{self} \neq), (C.\overset{n}{=}), (C.=), (C.\mu =)\}$$

$$C^2 = \{(C.\mu_n), (C.DD^0), (C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset), (C.EV^0), (C.U_n), \\ (C.U^0), (C.\text{self} \neq), (C.\frac{n}{\sim}), (C.=), (C.\mu=)\}$$

$$C^3 = \{(C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset), (C.E_n), (C.U_n), (C.\text{self} \neq), \\ (C.\frac{n}{\sim}), (C.=), (C.\mu=), (C.=)\}$$

$$C^4 = \{(C.\mu_n), (C.DD), (C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset), (C.E_n), (C.U_n), \\ (C.\text{self} \neq), (C.\frac{n}{\sim}), (C.=), (C.\mu=)\}$$

To show in these systems that a formula , \underline{A} , is self-sustaining we adopt a reductio ad absurdum procedure. We assume that the negation of the formula is a member of an arbitrary μ_n . If such an assumption leads to a contradiction of the rules in the appropriate C^i ($1 \leq i \leq 4$) , then unless the formula is prevented by $(C.\mu_n)$ from being satisfied, the formula is self-sustaining.

For example, to show that $(p \supset (q \supset p))$ is self-sustaining in

$\mu^4 Q = :$

- | | | |
|------------------------------------|---------|-----------------------------|
| 1. $\sim(p \supset (q \supset p))$ | μ_n | Assumption |
| 2. p | μ_n | by $(C.\sim\supset)$ from 1 |
| 3. $\sim(q \supset p)$ | μ_n | by $(C.\sim\supset)$ from 1 |
| 4. q | μ_n | by $(C.\sim\supset)$ from 3 |
| 5. $\sim p$ | μ_n | by $(C.\sim\supset)$ from 3 |

but by 2 and 5 $(C.\sim)$ is contradicted.

Since $(C.\mu_n)$ is not contradicted by 1 it follows that

$(p \supset (q \supset p))$ is self-sustaining.

Now consider $((U_x)F_x \supset Fa_1^2)$ in $\mu^4 Q =$.

There are two possibilities:

- (a) 1. $\sim((U_x)F_x \supset Fa_1^2)$ μ_n ($n \neq 2$) Assumption
but this contradicts $(C.\mu_n)$
- (b) 1. $\sim((U_x)F_x \supset Fa_1^2)$ μ_2 Assumption
2. $(U_x)F_x$ μ_2 by $(C.\sim\supset)$ from 1
3. $\sim Fa_1^2$ μ_2 by $(C.\sim\supset)$ from 1

4. $\text{Fa}_1^2 \in \mu_2$ by $(C.U_n)$ from 2,3.

which contradicts $(C.\sim)$.

The formula can belong to μ_2 so it is self-sustaining.

The systems $M^iQ = (1 \leq i \leq 4)$

The primitive symbols for these systems are the same as those for the respective $\mu^iQ = (1 \leq i \leq 4)$ as set out above with the addition of the improper symbol ' \Box ' in each system.

The formation rules for $M^iQ =$ are as for $\mu^iQ = (1 \leq i \leq 3)$ with the addition of

7. If \underline{A} is a wff so is $\Box \underline{A}$.

The formation rules for $M^4Q =$ are as for $\mu^4Q =$ with the addition of 7 and with 6 replaced by

6M. If \underline{A} is a wff and \underline{x} is any individual variable and \underline{a} is any individual constant, then, provided that all the individual constants in \underline{A} have the same superscript and provided that no modal operators occur in \underline{A} ,

$$(\underline{Ux})(\underline{A}(\underline{x}/\underline{a}))$$

is a wff, where $(\underline{A}(\underline{x}/\underline{a}))$ is the result of substituting

\underline{x} for \underline{a} at zero or more occurrences of \underline{a} in \underline{A} .

We define \Diamond as $\sim \Box \sim$.

We denote C^{M^iQ} by ' C_M^i ' ($1 \leq i \leq 4$)

We have the additional rules to draw from for the C_M^i ($1 \leq i \leq 4$):

$(C.\mu_n^\Diamond)$ If \underline{A} contains any individual constant \underline{a}^i occurring outside the scope of a modal operator and outside of an identity and $i \neq n$, then $\underline{A} \notin \mu_n$.

$(C.\Diamond^*)$ If $\Diamond \underline{A} \in \mu_n \in \Omega$ then there is in Ω at least one model set, such as μ_m , such that $\underline{A} \in \mu_m$ and μ_m is an

alternative to μ_n in Ω .

(C.□) If $\Box A \in \mu_n \in \Omega$ then $A \in \mu_n$.

(C.□□*) If $\Box A \in \mu_n \in \Omega$ then if μ_m is an alternative to μ_n in Ω then $\Box A \in \mu_m$.

(C.□□*) If $\Box A \in \mu_n \in \Omega$ and μ_n is an alternative to μ_m in Ω then $\Box A \in \mu_m$.

(C.□□i) If $\Box A \in \mu_n \in \Omega$ and μ_m is an alternative to μ_n in Ω or μ_n is an alternative to μ_m in Ω , then if A contains occurrences of the k individual constants

$\underline{a}_{i_1}^n, \dots, \underline{a}_{j_k}^n$ then for some k individual constants $\underline{b}_{x_1}^m, \dots, \underline{b}_{y_k}^m$: $\underline{a}_{i_1}^n = \underline{b}_{x_1}^m \in \mu_m, \dots, \underline{a}_{j_k}^n = \underline{b}_{y_k}^m \in \mu_m$.

Let $M = \{(C. \Diamond*), (C. \Box), (C. \Box \Box*), (C. \Box \Box*)\}$ then

$$C_M^1 = C^1 \cup M$$

$$C_M^2 = (C^2 - \{(C. \mu_n)\}) \cup \{(C. \mu_n^\Diamond)\} \cup M$$

$$C_M^3 = C^3 \cup M \cup \{(C. \Box \Box i)\}$$

$$C_M^4 = (C^4 - \{(C. \mu_n)\}) \cup \{(C. \mu_n^\Diamond)\} \cup M$$

We can now show that

$$(1) \quad \underline{a}^i = \underline{b}^j \equiv \Box(\underline{a}^i = \underline{b}^j)$$

is self-sustaining in all the systems $M^i Q = (1 \leq i \leq 4)$

$$1. \quad \underline{a}^i = \underline{b}^j \neq \Box(\underline{a}^i = \underline{b}^j) \in \mu_n \text{ Assumption}$$

so either:

$$2a. \quad \underline{a}^i = \underline{b}^j \in \mu_n$$

$$3a. \quad \sim \Box(\underline{a}^i = \underline{b}^j) \in \mu_n$$

$$4a. \quad \Diamond \sim (\underline{a}^i = \underline{b}^j) \in \mu_n \text{ from 3a by def. } \Diamond$$

$$5a. \quad \underline{a}^i \neq \underline{b}^j \in \mu_m \text{ from 4a by (C. } \Diamond *)$$

$$6a. \quad \underline{a}^i = \underline{b}^j \in \mu_m \text{ from 2a and 5a by (C. } \mu =)$$

which is contradictory;

or:

$$2b. \quad \underline{a}^i \neq \underline{b}^j \quad \in \mu_n$$

$$3b. \quad \Box(\underline{a}^i = \underline{b}^j) \quad \in \mu_n$$

$$4b. \quad \underline{a}^i = \underline{b}^j \quad \in \mu_n \text{ from 3b by (C.}\Box\text{)}$$

which is contradictory.

All the rules cited are common to all systems, so (1) is self-sustaining because, since all its atomic parts are identities, it is satisfiable in all systems. We can also show that

$$(2) \quad \underline{a}^i \neq \underline{b}^j \equiv \Box(\underline{a}^i \neq \underline{b}^j)$$

is self-sustaining in all systems.

$$1. \quad \underline{a}^i \neq \underline{b}^j \neq \Box(\underline{a}^i \neq \underline{b}^j) \quad \in \mu_n \text{ Assumption}$$

so either:

$$2a. \quad \underline{a}^i \neq \underline{b}^j \quad \in \mu_n$$

$$3a. \quad \sim \Box(\underline{a}^i \neq \underline{b}^j) \quad \in \mu_n$$

$$4a. \quad \Diamond \sim(\underline{a}^i \neq \underline{b}^j) \quad \in \mu_n \text{ def. } \Diamond$$

$$5a. \quad \underline{a}^i = \underline{b}^j \quad \in \mu_m \text{ from 4a by (C.}\Diamond\text{*)}$$

$$6a. \quad \underline{a}^i \neq \underline{b}^j \quad \in \mu_m \text{ from 2a and 5a by (C.}\mu=\text{)}$$

which is contradictory;

or:

$$2b. \quad \underline{a}^i = \underline{b}^j \quad \in \mu_n$$

$$3b. \quad \Box(\underline{a}^i \neq \underline{b}^j) \quad \in \mu_n$$

$$4b. \quad \underline{a}^i \neq \underline{b}^j \quad \in \mu_n \text{ from 3b by (C.}\Box\text{)}$$

which is contradictory.

(2), like (1), is satisfiable in all systems and is therefore self-sustaining.

In both $M^2Q=$ and $M^3Q=$ the formula

$$(Ux)(\underline{F}a^i \supset \underline{F}a^i)$$

is self-sustaining where $i = 0$ for $M^2Q=$ and $i \geq 1$ for $M^3Q=$.

1. $\sim \Box (Ux)(Fa^i \supset Fa^i)$ $\in \mu_n$ Assumption
2. $\Diamond \sim (Ux)(Fa^i \supset Fa^i)$ $\in \mu_n$ from 1 by def. \Diamond
3. $\sim (Ux)(Fa^i \supset Fa^i)$ $\in \mu_m$ from 2 by (C. \Diamond^*)
4. $(Ex)\sim (Fa^i \supset Fa^i)$ $\in \mu_m$ from 3 by def. E
5. $\sim (Fa^i \supset Fa^i)$ $\in \mu_m$ from 4 by (C.E⁰) or (C.E)
6. $Fa^i \ \& \ \sim Fa^i$ $\in \mu_m$ from 5 by def. &

which is contradictory.

Since there is no rule to prevent the formula being satisfiable in some μ_n it is self-sustaining.

For the Barcan Formula and its converse in $M^1Q = :-$

1. $\sim ((Ux)\Box Fx \supset \Box (Ux)Fx)$ $\in \mu_n$ Assumption
2. $(Ux)\Box Fx$ $\in \mu_n$ 1 (C. $\sim \supset$)
3. $\sim \Box (Ux)Fx$ $\in \mu_n$ 1 (C. $\sim \supset$)
4. $\Diamond \sim (Ux)Fx$ $\in \mu_n$ 3 def. \Diamond
5. $\sim (Ux)Fx$ $\in \mu_m$ 4 (C. \Diamond^*)
6. $(Ex)\sim Fx$ $\in \mu_m$ 5 def. E
7. $\sim Fa$ $\in \mu_m$ 6 (C.E)
8. $\Box Fa$ $\in \mu_n$ 2 (C.U)
9. $\Box Fa$ $\in \mu_m$ 8 (C. $\Box \Box^*$)
10. Fa $\in \mu_m$ 9 (C. \Box)

which is contradictory.

The Barcan Formula is self-sustaining in $M^1Q =$.

For its converse:-

1. $\Box (Ux)Fx$ $\in \mu_n$ } Assumption
2. $\sim (Ux)\Box Fx$ $\in \mu_n$ }
3. $(Ex)\sim \Box Fx$ $\in \mu_n$ 2 def. E
4. $\sim \Box Fa$ $\in \mu_n$ 3 (C.E)
5. $\Diamond \sim Fa$ $\in \mu_n$ 4 def. \Diamond
6. $\sim Fa$ $\in \mu_m$ 5 (C. \Diamond^*)

7. $\Box (Ux)Fx$ $\in \mu_m 1,6 (C.\Box\Box^*)$
8. $(Ux)Fx$ $\in \mu_m 7 (C.\Box)$
9. Fa $\in \mu_m 8 (C.U)$

which is contradictory.

The converse of the Barcan Formula is self-sustaining in $M^1Q =$.

So $(Ux)\Box Fx \equiv \Box(Ux)Fx$ is self-sustaining.

For $(Ex)\Box Fx \equiv \Box(Ex)Fx$ in $M^1Q = :-$

1. $(Ex)\Box Fx$ $\in \mu_n \left. \begin{array}{l} \text{Assumption} \end{array} \right\}$
2. $\sim \Box(Ex)Fx$ $\in \mu_n$
3. $\Diamond \sim(Ex)Fx$ $\in \mu_n 2 \text{ def. } \Diamond$
4. $\Box Fa$ $\in \mu_n 1 (C.E)$
5. $(Ux)\sim Fx$ $\in \mu_m 3 (C.\Diamond^*)$
6. $\Box Fa$ $\in \mu_m 4 (C.\Box\Box^*)$
7. Fa $\in \mu_m 6 (C.\Box)$
8. $\sim Fa$ $\in \mu_m 5 (C.U)$

which is contradictory.

1. $\Box(Ex)Fx$ $\in \mu_n \text{ Assumption}$
2. $\sim(Ex)\Box Fx$ $\in \mu_n$
3. $(Ux)\sim \Box Fx$ $\in \mu_n 2 \text{ def. } E$
4. $(Ex)Fx$ $\in \mu_n 1 (C.\Box)$
5. Fa $\in \mu_n 4 (C.E)$
6. $\sim \Box Fa$ $\in \mu_n 3 (C.U)$
7. $\Diamond \sim Fa$ $\in \mu_n 6 \text{ def. } \Diamond$
8. $\sim Fa$ $\in \mu_m 7 (C.\Diamond^*)$
9. $(Ex)Fx$ $\in \mu_m 1 (C.\Box\Box^*)$

which is not contradictory.

So although $(Ex)\Box Fx \supset \Box(Ex)Fx$ is self-sustaining,

the converse is not. A counter-model for the converse, i.e. a

model system in which the negation of the converse is satisfiable

is:

$$\Omega = \{\mu_1, \mu_2\} \quad \text{where}$$

$$\mu_1 = \{\Box(E\underline{x})F\underline{x}, \sim(E\underline{x})\Box F\underline{x}, (E\underline{x})F\underline{x}, Fa^1, \sim\Box Fa^1\}$$

$$\mu_2 = \{\Box(E\underline{x})F\underline{x}, (E\underline{x})F\underline{x}, \sim Fa^1, Fa^2\}$$

For $(U\underline{x})\Box(\underline{x} = \underline{a}) \equiv \Box(U\underline{x})(\underline{x} = \underline{a})$ in $M^1Q = :-$

- | | | |
|--|-------------|----------------------------------|
| 1. $(U\underline{x})\Box(\underline{x} = \underline{a})$ | $\in \mu_n$ | } Assumption |
| 2. $\sim\Box(U\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_n$ | |
| 3. $\Diamond\sim(U\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_n$ | 2 def. |
| 4. $\Box(\underline{a} = \underline{a})$ | $\in \mu_n$ | 1 (C.U) |
| 5. $(E\underline{x})\sim(\underline{x} = \underline{a})$ | $\in \mu_m$ | 3 (C. \Diamond^*) def. E |
| 6. $(\underline{b} \neq \underline{a})$ | $\in \mu_m$ | 5 (C.E) |
| 7. $\Box(\underline{b} = \underline{a})$ | $\in \mu_n$ | 1 (C.U) |
| 8. $(\underline{b} = \underline{a})$ | $\in \mu_m$ | 7 (C. $\Box\Box^*$)(C. \Box) |

which is contradictory.

- | | | |
|--|-------------|----------------------------------|
| 1. $\Box(U\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_n$ | } Assumption |
| 2. $\sim(U\underline{x})\Box(\underline{x} = \underline{a})$ | $\in \mu_n$ | |
| 3. $(E\underline{x})\sim\Box(\underline{x} = \underline{a})$ | $\in \mu_n$ | 2 def.E |
| 4. $\Diamond\sim(\underline{b} = \underline{a})$ | $\in \mu_n$ | 3 (C.E) def. \Diamond |
| 5. $(\underline{b} \neq \underline{a})$ | $\in \mu_m$ | 4 (C. \Diamond^*) |
| 6. $(U\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_m$ | 1 (C. $\Box\Box^*$)(C. \Box) |
| 7. $(\underline{b} = \underline{a})$ | $\in \mu_m$ | 6 (C.U) |

which is contradictory.

So the equivalence tested is self-sustaining in M^1Q .

For $(E\underline{x})\Box(\underline{x} = \underline{a}) \equiv \Box(E\underline{x})(\underline{x} = \underline{a})$ in $M^1Q = :-$

- | | | |
|--|-------------|----------------------|
| 1. $(E\underline{x})\Box(\underline{x} = \underline{a})$ | $\in \mu_n$ | } Assumption |
| 2. $\sim\Box(E\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_n$ | |
| 3. $\Diamond\sim(E\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_n$ | 2 def. \Diamond |
| 4. $\Box(\underline{b} = \underline{a})$ | $\in \mu_n$ | 1 (C.E) |
| 5. $\sim(E\underline{x})(\underline{x} = \underline{a})$ | $\in \mu_m$ | 3 (C. \Diamond^*) |

- | | | | |
|----|------------------|-------------|----------------------------|
| 6. | $(Ux)(x \neq a)$ | $\in \mu_m$ | 5 def.E |
| 7. | $(b = a)$ | $\in \mu_m$ | 4 $(C.\Box\Box^*)(C.\Box)$ |
| 8. | $(b \neq a)$ | $\in \mu_m$ | 6 $(C.U)$ |

which is contradictory.

- | | | | |
|----|-----------------------|-------------|---------------------------|
| 1. | $\Box(Ex)(x = a)$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim(Ex)\Box(x = a)$ | $\in \mu_n$ | |
| 3. | $(Ux)\sim\Box(x = a)$ | $\in \mu_n$ | 2 def.E |
| 4. | $\Diamond(a \neq a)$ | $\in \mu_n$ | 3 $(C.U)$ def. \Diamond |
| 5. | $(a \neq a)$ | $\in \mu_m$ | 4 $(C.\Diamond^*)$ |

which is contradictory.

So the equivalence tested is self-sustaining in $M^1Q =$.

For the Barcan Formula and its converse in $M^3Q =$:-

- | | | | |
|----|----------------------|-------------|----------------------------|
| 1. | $(Ux)\Box Fx$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim\Box(Ux)Fx$ | $\in \mu_n$ | |
| 3. | $\Diamond\sim(Ux)Fx$ | $\in \mu_n$ | 2 def. \Diamond |
| 4. | $\sim(Ux)Fx$ | $\in \mu_m$ | 3 $(C.\Diamond^*)$ |
| 5. | $(Ex)\sim Fx$ | $\in \mu_m$ | 4 def.E |
| 6. | $\sim Fa^m$ | $\in \mu_m$ | 5 $(C.E_n)$ |
| 7. | $\Box Fa^n$ | $\in \mu_n$ | 1 $(C.U_n)$ |
| 8. | Fa^n | $\in \mu_m$ | 7 $(C.\Box\Box^*)(C.\Box)$ |
| 9. | $(a^n = b^m)$ | $\in \mu_m$ | 7 $(C.\Box\Box i)$ |

which is not contradictory.

- | | | | |
|----|---------------------|-------------|-----------------------------|
| 1. | $\Box(Ux)Fx$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim(Ux)\Box Fx$ | $\in \mu_n$ | |
| 3. | $(Ex)\sim\Box Fx$ | $\in \mu_n$ | 2 def.E |
| 4. | $\Diamond\sim Fa^n$ | $\in \mu_n$ | 3 $(C.E_n)$ def. \Diamond |
| 5. | $\sim Fa^n$ | $\in \mu_m$ | 4 $(C.\Diamond^*)$ |
| 6. | $(Ux)Fx$ | $\in \mu_m$ | 1 $(C.\Box\Box^*)(C.\Box)$ |
| 7. | Fa^m | $\in \mu_m$ | 6 $(C.U_n)$ |

which is not contradictory.

So the Barcan Formula is not self-sustaining, nor is its converse self-sustaining in $M^3Q =$.

For $(\underline{E}\underline{x})\Box F\underline{x} \equiv \Box(\underline{E}\underline{x})F\underline{x}$ in $M^3Q = :-$

- | | | | |
|-----|--|-------------|---------------------------------|
| 1. | $(\underline{E}\underline{x})\Box F\underline{x}$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim \Box(\underline{E}\underline{x})F\underline{x}$ | $\in \mu_n$ | |
| 3. | $\Diamond \sim (\underline{E}\underline{x})F\underline{x}$ | $\in \mu_n$ | 2 def. \Diamond |
| 4. | $\Box Fa^n$ | $\in \mu_n$ | 1 (C. E_n) |
| 5. | $\sim (\underline{E}\underline{x})F\underline{x}$ | $\in \mu_m$ | 3 (C. $\Diamond *$) |
| 6. | Fa^n | $\in \mu_m$ | 4 (C. $\Box\Box*$)(C. \Box) |
| 7. | $(\underline{a}^n = \underline{b}^m)$ | $\in \mu_m$ | 4 (C. $\Box\Box i$) |
| 8. | $(\underline{U}\underline{x})\sim F\underline{x}$ | $\in \mu_m$ | 5 def. E |
| 9. | $\sim Fb^m$ | $\in \mu_m$ | 8 (C. U_n) |
| 10. | $\sim Fa^n$ | $\in \mu_m$ | 7, 9 (C. $=$) |

which is contradictory.

- | | | | |
|----|--|-------------|---------------------------------|
| 1. | $\Box(\underline{E}\underline{x})F\underline{x}$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim (\underline{E}\underline{x})\Box F\underline{x}$ | $\in \mu_n$ | |
| 3. | $(\underline{U}\underline{x})\sim \Box F\underline{x}$ | $\in \mu_n$ | 2 def. E |
| 4. | $(\underline{E}\underline{x})F\underline{x}$ | $\in \mu_n$ | 1 (C. \Box) |
| 5. | Fa^n | $\in \mu_n$ | 4 (C. E_n) |
| 6. | $\Diamond \sim Fa^n$ | $\in \mu_n$ | 3 (C. U_n) def. \Diamond |
| 7. | $\sim Fa^n$ | $\in \mu_m$ | 6 (C. $\Diamond *$) |
| 8. | $(\underline{E}\underline{x})F\underline{x}$ | $\in \mu_m$ | 1 (C. $\Box\Box*$)(C. \Box) |
| 9. | Fb^m | $\in \mu_m$ | 8 (C. E_n) |

which is not contradictory.

So $(\underline{E}\underline{x})\Box F\underline{x} \supset \Box(\underline{E}\underline{x})F\underline{x}$ is self-sustaining in $M^3Q =$, but its converse is not.

For $(Ux) \Box (x = a) \equiv \Box (Ux)(x = a)$ in $M^3Q = :-$

- | | | | |
|----|-------------------------------|-------------|------------------------------------|
| 1. | $(Ux) \Box (x = a^n)$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim \Box (Ux)(x = a^n)$ | $\in \mu_n$ | |
| 3. | $\Diamond \sim (Ux)(x = a^n)$ | $\in \mu_n$ | 2 def. \Diamond |
| 4. | $(Ex)(x \neq a^n)$ | $\in \mu_m$ | 3 (C. \Diamond^*) def. E |
| 5. | $(a^n \neq b^m)$ | $\in \mu_m$ | 4 (C. E_n) |
| 6. | $\Box (b^n = a^n)$ | $\in \mu_n$ | 1 (C. U_n) |
| 7. | $(b^n = a^n)$ | $\in \mu_m$ | 6 (C. $\Box \Box^*$) (C. \Box) |

which is not contradictory.

- | | | | |
|----|------------------------------|-------------|------------------------------------|
| 1. | $\Box (Ux)(x = a^n)$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim (Ux) \Box (x = a^n)$ | $\in \mu_n$ | |
| 3. | $(Ex) \Diamond (x \neq a^n)$ | $\in \mu_n$ | 2 def. E def. \Diamond |
| 4. | $\Diamond (b^n \neq a^n)$ | $\in \mu_n$ | 3 (C. E_n) |
| 5. | $(b^n \neq a^n)$ | $\in \mu_m$ | 4 (C. \Diamond^*) |
| 6. | $(Ux)(x = a^n)$ | $\in \mu_m$ | 1 (C. $\Box \Box^*$) (C. \Box) |
| 7. | $(a^n = a^m)$ | $\in \mu_m$ | 1 (C. $\Box \Box i$) |
| 8. | $(a^m = a^n)$ | $\in \mu_m$ | 6 (C. U_n) |

which is not contradictory.

So the equivalence tested is not self-sustaining.

For the formula $(Ex) \Box (x = a^n) \equiv \Box (Ex)(x = a^n)$ in $M^3Q = :-$

- | | | | |
|----|-----------------------------|-------------|------------------------------------|
| 1. | $(Ex) \Box (x = a^n)$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim \Box (Ex)(x = a^n)$ | $\in \mu_n$ | |
| 3. | $\Box (b^n = a^n)$ | $\in \mu_n$ | 1 (C. E_n) |
| 4. | $\Diamond (Ux)(x \neq a^n)$ | $\in \mu_n$ | 2 def. E def. \Diamond |
| 5. | $(Ux)(x \neq a^n)$ | $\in \mu_m$ | 4 (C. \Diamond^*) |
| 6. | $(b^n = a^n)$ | $\in \mu_m$ | 3 (C. $\Box \Box^*$) (C. \Box) |
| 7. | $(a^n = b^m)$ | $\in \mu_m$ | 3, 5 (C. $\Box \Box i$) |
| 8. | $(b^m \neq a^n)$ | $\in \mu_m$ | 5 (C. U_n) |

which is contradictory.

- | | | | |
|----|--|-------------|-------------------------|
| 1. | $\Box(\underline{Ex})(\underline{x} = \underline{a}^n)$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim(\underline{Ex})\Box(\underline{x} = \underline{a}^n)$ | $\in \mu_n$ | |
| 3. | $(\underline{Ex})(\underline{x} = \underline{a}^n)$ | $\in \mu_n$ | 1 (C. \Box) |
| 4. | $(\underline{b}^n = \underline{a}^n)$ | $\in \mu_n$ | 3 (C. E_n) |
| 5. | $(\underline{Ux})\Diamond(\underline{x} \neq \underline{a}^n)$ | $\in \mu_n$ | 2 def.E def. \Diamond |
| 6. | $\Diamond(\underline{b}^n \neq \underline{a}^n)$ | $\in \mu_n$ | 5 (C. U_n) |
| 7. | $(\underline{b}^n \neq \underline{a}^n)$ | $\in \mu_m$ | 6 (C. \Diamond^*) |
| 8. | $(\underline{b}^n = \underline{a}^n)$ | $\in \mu_m$ | 4 (C. $\mu =$) |

which is contradictory.

So the equivalence tested is self-sustaining.

Since in $M^4Q =$ there are no formulae of the form $(\underline{Ex})\underline{OA}(\underline{x})$ nor of the form $(\underline{Ux})\underline{OA}(\underline{x})$ where O is either \Box or \Diamond , and $\underline{A}(\underline{x})$ means that \underline{A} contains \underline{x} , there is no point in looking at any of the equivalences we have considered above in terms of $M^4Q =$.

We therefore test more formulae of interest in $M^1Q =$ and $M^3Q =$.

(i) $(\underline{Ux})(\underline{Uy})(\underline{x} = \underline{y} \ \& \ F\underline{x} \ \supset \ F\underline{y})$

- | | | | |
|----|--|-------------|-------------------------------|
| 1. | $\sim(\underline{Ux})(\underline{Uy})(\underline{x} = \underline{y} \ \& \ F\underline{x} \ \supset \ F\underline{y})$ | $\in \mu_n$ | Assumption |
| 2. | $(\underline{Ex})(\underline{Ey})(\underline{x} = \underline{y} \ \& \ F\underline{x} \ \& \ \sim F\underline{y})$ | $\in \mu_n$ | 1 def.E def.& |
| 3. | $(\underline{Ey})(\underline{a}^n = \underline{y} \ \& \ F\underline{a}^n \ \& \ \sim F\underline{y})$ | $\in \mu_n$ | 2 (C. E_n) |
| 4. | $\underline{a}^n = \underline{b}^n \ \& \ F\underline{a}^n \ \& \ \sim F\underline{b}^n$ | $\in \mu_n$ | 3 (C. E_n) |
| 5. | $F\underline{a}^n$ | $\in \mu_n$ | 4 def.& |
| 6. | $\sim F\underline{a}^n$ | $\in \mu_n$ | 4 def.& (C. \underline{n}) |

which is contradictory, so (i) is self-sustaining.

(ii) $(\underline{Ux})(\underline{Uy})(\underline{x} = \underline{y} \ \supset \ \Box(\underline{x} = \underline{y}))$

- | | | | |
|----|---|-------------|---------------|
| 1. | $\sim(\underline{Ux})(\underline{Uy})(\underline{x} = \underline{y} \ \supset \ \Box(\underline{x} = \underline{y}))$ | $\in \mu_n$ | Assumption |
| 2. | $(\underline{Ex})(\underline{Ey})(\underline{x} = \underline{y} \ \& \ \sim \Box(\underline{x} = \underline{y}))$ | $\in \mu_n$ | 1 def.E def.& |
| 3. | $\underline{a}^n = \underline{b}^n \ \& \ \sim \Box(\underline{a}^n = \underline{b}^n)$ | $\in \mu_n$ | 2 (C. E_n) |

4. $\underline{a}^n = \underline{b}^n$ $\in \mu_n$ 3 def.&
5. $\Diamond \sim (\underline{a}^n = \underline{b}^n)$ $\in \mu_n$ 3 def.& def. \Diamond
6. $\sim (\underline{a}^n = \underline{b}^n)$ $\in \mu_m$ 5 (C. \Diamond^*)
7. $\underline{a}^n = \underline{b}^n$ $\in \mu_m$ 4,6, (C. $\mu =$)

which is contradictory, so (ii) is self-sustaining.

(iii) $(U\underline{x})(U\underline{y})(F\underline{x}\underline{y} \supset \Box F\underline{x}\underline{y})$

1. $\sim (U\underline{x})(U\underline{y})(F\underline{x}\underline{y} \supset \Box F\underline{x}\underline{y})$ $\in \mu_n$ Assumption
2. $(E\underline{x})(E\underline{y})(F\underline{x}\underline{y} \& \sim \Box F\underline{x}\underline{y})$ $\in \mu_n$ 1 def.E def.&
3. $F\underline{a}^n \underline{b}^n \& \sim \Box F\underline{a}^n \underline{b}^n$ $\in \mu_n$ 2 (C. E_n)
4. $F\underline{a}^n \underline{b}^n$ $\in \mu_n$ 3 def.&
5. $\Diamond \sim F\underline{a}^n \underline{b}^n$ $\in \mu_n$ 3 def.&
6. $\sim F\underline{a}^n \underline{b}^n$ $\in \mu_m$ 5 (C. \Diamond^*)

which is not contradictory, so (iii) is not self-sustaining.

(iv) $\Box (\underline{a}^n = \underline{a}^n) \supset (E\underline{x})\Box (\underline{x} = \underline{a}^n)$

1. $\Box (\underline{a}^n = \underline{a}^n)$ $\in \mu_m$ } Assumption
2. $\sim (E\underline{x})\Box (\underline{x} = \underline{a}^n)$ $\in \mu_m$ }
3. $(U\underline{x})\Diamond (\underline{x} \neq \underline{a}^n)$ $\in \mu_m$ 2 def.E def. \Diamond
4. $\Diamond (\underline{a}^m \neq \underline{a}^n)$ $\in \mu_m$ 3 (C. U_n)
5. $(\underline{a}^m \neq \underline{a}^n)$ $\in \mu_i$ 4 (C. \Diamond^*)
6. $(\underline{a}^n \neq \underline{a}^n)$ $\in \mu_i$ 1 (C. $\Box\Box^*$)(C. \Box)

which is not contradictory, so (iv) is not self-sustaining.

(xi) $(E!\underline{x})\Box \underline{A} \supset (E!\underline{x})\underline{A}$

1. $(E\underline{y})(U\underline{x})(\Box \underline{A} \equiv \underline{x} = \underline{y})$ $\in \mu_n$ } Assumption
2. $\sim (E\underline{y})(U\underline{x})(\underline{A} \equiv \underline{x} = \underline{y})$ $\in \mu_n$ }
3. $(U\underline{y})(E\underline{x})(\underline{A} \neq \underline{x} = \underline{y})$ $\in \mu_n$ 2 def.E def. \Diamond
4. $(U\underline{x})(\Box \underline{A} \equiv \underline{x} = \underline{a}^n)$ $\in \mu_n$ 1 (C. E_n)
5. $(E\underline{x})(\underline{A} \neq \underline{x} = \underline{a}^n)$ $\in \mu_n$ 3 (C. U_n)
6. $\underline{A} \neq \underline{b}^n = \underline{a}^n$ $\in \mu_n$ 5 (C. E_n)
7. $\Box \underline{A} \equiv \underline{b}^n = \underline{a}^n$ $\in \mu_n$ 4 (C. U_n)

which is not contradictory, so (xi) is not self-sustaining.

Without setting out the system M^3QI' , but relying on the rules and other details as set out in the main body of the text (pp. we test formula (xii) in M^3QI' .

- | | | | |
|-----|---|-------------|---|
| 1. | $(\underline{I}_Y)[N_Y, (\underline{I}_X)[N_X, x = Y]]$ | $\in \mu_n$ | Assumption |
| 2. | $\sim(E_X) \square (\underline{I}_Y)[N_Y, x = Y]$ | $\in \mu_n$ | |
| 3. | $(U_X) \diamond \sim(\underline{I}_Y)[N_Y, x = Y]$ | $\in \mu_n$ | 2 def.E def. \diamond |
| 4. | $\sim(\underline{I}_Y)[N_Y, \underline{a}^n = Y]$ | $\in \mu_n$ | 3 (C.U _n) |
| 5. | $\sim(\underline{I}_Y)[N_Y, \underline{a}^n = Y]$ | $\in \mu_m$ | 4 (C. \diamond^*) |
| 6. | $(U_Y)(N_Y \& \underline{a}^n = Y \supset (E_Z)(N_Z \equiv Y \neq Z))$ | $\in \mu_m$ | 5 (C. $\sim I$) |
| 7. | $(\underline{I}_Y)[N_Y, (\underline{I}_X)[N_X, x = Y]]$ | $\in \mu_m$ | 1 (C. $\square \square^*$) (C. \square) |
| 8. | $N_{\underline{a}}^m$ | $\in \mu_m$ | 7 (C. I _n) |
| 9. | $N_{\underline{a}}^m \& \underline{a}^n = \underline{a}^m \supset (E_Z)(N_Z \equiv \underline{a}^m \neq Z)$ | $\in \mu_m$ | 6 (C.U _n) |
| 10. | $\underline{a}^n = \underline{a}^m \supset (E_Z)(N_Z \equiv \underline{a}^m \neq Z)$ | $\in \mu_m$ | 8,9 |

which is not contradictory, so (xii) is not self-sustaining.

The systems M^i_{QF} ($1 \leq i \leq 4$)

The systems M^i_{QF} have the same symbols as $M^i_Q =$ ($1 \leq i \leq 4$) respectively but with the addition of the two improper symbols :

$$\Pi \quad \Sigma$$

The formation rule to be added to the rules for $M^i_Q =$ ($2 \leq i \leq 3$) is:

- 6F. If \underline{A} is a wff and \underline{x} is any individual variable, then, provided that all the individual constants in \underline{A} have the same superscript,

$$(\Pi \underline{x})(\underline{A}(\underline{x} // \underline{a}))$$

is a wff.

The formation rule to be added to the rules for $M^1_Q =$ is:

- 6'F. If \underline{A} is a wff and \underline{x} is any individual variable then

$$(\Pi \underline{x})(\underline{A}(\underline{x} // \underline{a}))$$

is a wff.

The formation rule to be added to the rules for $M^4Q=$ is:

6MF. If \underline{A} is a wff and \underline{x} is any individual variable and \underline{a} is any individual constant, then provided that all the individual constants in \underline{A} have the same superscript and provided that no modal operators occur in \underline{A} ,

$$(\Pi \underline{x})(\underline{A}(\underline{x}/\underline{a}))$$

is a wff.

We also define: $(\Sigma \underline{x})\underline{A} \quad \text{df} \quad \sim(\Pi \underline{x})\sim \underline{A}$

For these systems we denote the sets C^{M^1QF} by ' C_F^i ' ($1 \leq i \leq 4$) respectively. The sets C_F^i ($1 \leq i \leq 4$) are made up as follows: first we replace each occurrence of 'E' with ' Σ ' and each occurrence of 'U' with ' Π ' in each of the sets C_M^i ($1 \leq i \leq 4$) to give the sets C_{F*}^i ($1 \leq i \leq 4$) respectively; then to these sets are added the rules set out below as indicated.

We add to each C_{F*}^i ($1 \leq i \leq 4$) the rules:

(C.E Σ) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then $(\Sigma \underline{x})\underline{A} \in \mu_n$.

(C. Π U) If $(\Pi \underline{x})\underline{A} \in \mu_n \in \Omega$ then $(U\underline{x})\underline{A} \in \mu_n$.

to give the sets C_{f*}^i ($1 \leq i \leq 4$) respectively.

The set C_F^1 consists of the set C_{f*}^1 together with the rules:

(C.E $_0$) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then $(E\underline{x})(\underline{x} = \underline{a}) \in \mu_n$ and $(\underline{A}(\underline{a}/\underline{x})) \in \mu_n$ for individual constant \underline{a} .

(C.U $_0$) If $(U\underline{x})\underline{A} \in \mu_n \in \Omega$ and if $(E\underline{x})(\underline{x} = \underline{a}) \in \mu_n$ then $(\underline{A}(\underline{a}/\underline{x})) \in \mu_n$.

The set C_F^2 consists of the set C_{f*}^2 together with the rules:

(C.E $_2^0$) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then either $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$ and $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$ or $(E\underline{x})(\underline{x} = \underline{a}^0) \in \mu_n$ and $(\underline{A}(\underline{a}^0/\underline{x})) \in \mu_n$, for some individual constant \underline{a}^n or \underline{a}^0 .

(C.U⁰) If $(U\underline{x})\underline{A} \in \mu_n \in \Omega$ and if $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$
 then $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$.

The set C_F^3 consists of the set C_{F*}^3 together with the rules
 (C.U⁰) as above and

(C.E⁰) If $(E\underline{x})\underline{A} \in \mu_n \in \Omega$ then $(E\underline{x})(\underline{x} = \underline{a}^n) \in \mu_n$ and
 $(\underline{A}(\underline{a}^n/\underline{x})) \in \mu_n$ for some individual constant \underline{a}^n .

The set C_F^4 consists of the set C_{F*}^4 together with the rules (C.U⁰)
 and (C.E⁰) as set out above.

Consider the formulae

(xiii) $(U\underline{x})\Diamond \sim (E\underline{y})(\underline{y} = \underline{x})$ and

(xiv) $(U\underline{x})\Box \Diamond \sim (E\underline{y})(\underline{y} = \underline{x})$

in the systems M^i_{QF} ($1 \leq i \leq 4$)

We test to see if (xiii) is self-sustaining in M^1_{QF} .

1. $\sim (U\underline{x})\Diamond \sim (E\underline{y})(\underline{y} = \underline{x}) \in \mu_n$ Assumption
2. $(E\underline{x})\Box (E\underline{y})(\underline{y} = \underline{x}) \in \mu_n$ 1 def.E def. \Diamond (C. $\sim\sim$)
3. $\Box (E\underline{y})(\underline{y} = \underline{a}) \in \mu_n$ 2 (C.E⁰)
4. $(E\underline{y})(\underline{y} = \underline{a}) \in \mu_n$ 3 (C. \Box)

This will not lead to contradiction.

Similarly with (xiv). Neither are self-sustaining.

APPENDIX III

Epistemic Logics

In the text we indicated that in Hintikka's HS4 two new improper symbols 'K' and 'P' are added to the notation set out there for u^1Q . There are other differences in symbolism which should be noted. Individual constants and variables come in two categories, there are personal and impersonal individual constants and variables¹. Furthermore, individual constants have no superscripts. Because there are no free individual variables in HS4, just as there are none in our systems, what we call 'individual variables' Hintikka calls 'bound variables', and what we call 'individual constants' he calls 'free variables'.

In spite of this difference in terminology we can set out the primitive symbols for HS4 as:

Improper symbols:	$\sim \supset U E \underline{K} \underline{P} ()$
Propositional variables:	$p_1, q_1, r_1, p_2, q_2, r_2, p_3, \dots$
Individual variables (personal):	$x_1, y_1, z_1, x_2, y_2, z_2, x_3, \dots$
Individual variables (impersonal):	$t_1, u_1, v_1, t_2, u_2, v_2, t_3, \dots$
Individual constants (personal):	$a_1, b_1, c_1, a_2, b_2, c_2, a_3, \dots$
Individual constants (impersonal):	$i_1, j_1, k_1, i_2, j_2, k_2, i_3, \dots$
n-ary predicate variables:	$F_1^n, G_1^n, H_1^n, F_2^n, G_2^n, H_2^n, F_3^n, \dots (n \geq 1)$
predicate constant:	$=$

The formation rules are rules 1, 2, 3, 4 and 6' from pages 193-194 of Appendix II, together with:

5. If A and B are wffs then so is $(\underline{A} \supset \underline{B})$.
7. If A is a wff and a is any individual constant (personal) then $\underline{K}_a \underline{A}$ is a wff.

T. Terminal clause.

We define:

$$\underline{P_a A} =_{df} \sim \underline{K_a} \sim A$$

We retain the same definitions of model set, model system, satisfiable and self-sustaining as for the systems in Appendix II.

The set C^{HS4} is made up from the conditions $(C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset)$ from Appendix II, together with $(C.E_0), (C.U_0), (C.self \neq)$ and the following:

(C.K \Rightarrow) If $\underline{A} \in \mu_n$ and $(\underline{a} = \underline{b}) \in \mu_n$ and \underline{A} is an atomic formula or identity then $(\underline{A}(\underline{a}/\underline{b})) \in \mu_n$.

(C.=K) If $\underline{K_a A} \in \mu_n$ and $(\underline{a} = \underline{b}) \in \mu_n$ then $\underline{K_b A} \in \mu_n$.

(C.=P) If $\underline{P_a A} \in \mu_n$ and $(\underline{a} = \underline{b}) \in \mu_n$ then $\underline{P_b A} \in \mu_n$.

(C.P*) If $\underline{P_a A} \in \mu_n \in \Omega$ then there is in Ω at least one model set, such as μ_m , such that $\underline{A} \in \mu_m$ and μ_m is an alternative to μ_n with respect to \underline{a} in Ω .

(C.K) If $\underline{K_a A} \in \mu_n \in \Omega$ then $\underline{A} \in \mu_n$.

(C.KK*) If $\underline{K_a A} \in \mu_n \in \Omega$ and μ_m is an alternative to μ_n in Ω with respect to \underline{a} then $\underline{K_a A} \in \mu_m$.

Formulae:-

(i) A countermodel for $(U\underline{x})\underline{K_a Fx} \supset \underline{K_a}(U\underline{x})Fx$

would consist of the following sets μ_n and μ_m :

$$\mu_n = \{ (U\underline{x})\underline{K_a Fx}, \sim \underline{K_a}(U\underline{x})Fx, \underline{P_a}(E\underline{x})\sim Fx \}$$

$$\mu_m = \{ (E\underline{x})\sim Fx, (E\underline{x})(\underline{x} = \underline{b}), \sim F\underline{b} \}$$

Note: we cannot instantiate $(U\underline{x})\underline{K_a Fx}$ because there is no formula of the form $(E\underline{x})(\underline{x} = \underline{b})$ in μ_n .

(ii) A countermodel for $\underline{K_a}(U\underline{x})Fx \supset (U\underline{x})\underline{K_a Fx}$

would consist of the following sets μ_n and μ_m :

$$\mu_n = \{ \underline{K_a}(Ux)Fx, \sim(Ux)\underline{K_a}Fx, (Ex)\underline{P_a}\sim Fx, (Ex)(x = b), \underline{P_a}\sim Fb, \\ (Ux)Fx, Fb \}$$

$$\mu_m = \{ \sim Fb, \underline{K_a}(Ux)Fx, (Ux)Fx \}$$

Note: we cannot instantiate $(Ux)Fx$ in μ_m because there is no formula of the form $(Ex)(x = a)$ in μ_m .

(iii) A countermodel for $(Ex)\underline{K_a}Fx \supset \underline{K_a}(Ex)Fx$ consists of :

$$\mu_n = \{ (Ex)\underline{K_a}Fx, \sim\underline{K_a}(Ex)Fx, \underline{P_a}(Ux)\sim Fx, (Ex)(x = b), \underline{K_a}Fb \}$$

$$\mu_m = \{ (Ux)\sim Fx, \underline{K_a}Fb, Fb \}$$

(iv) A countermodel for $\underline{K_a}(Ex)Fx \supset (Ex)\underline{K_a}Fx$ consists of :

$$\mu_n = \{ \underline{K_a}(Ex)Fx, \sim(Ex)\underline{K_a}Fx, (Ux)\underline{P_a}\sim Fx, (Ex)Fx, (Ex)(x = b), \\ Fb, \underline{P_a}\sim Fb \}$$

$$\mu_m = \{ \sim Fb, \underline{K_a}(Ex)Fx, (Ex)Fx, (Ex)(x = c), Fc \}$$

(v) A countermodel for $(Ex)\underline{K_a}(b = x) \supset \underline{K_a}(Ex)(b = x)$ consists of :

$$\mu_n = \{ (Ex)\underline{K_a}(b = x), \sim\underline{K_a}(Ex)(b = x), (Ex)(b = c), \underline{K_a}(c = b), \\ \underline{P_a}(Ux)(b \neq x) \}$$

$$\mu_m = \{ (Ux)(b \neq x), \underline{K_a}(c = b) \}$$

(vi) A proof that $\underline{K_a}(Ex)(b = x) \supset (Ex)\underline{K_a}(b = x)$ is self-sustaining:

- | | | |
|-------------------------------------|-------------|---------------------------|
| 1. $\underline{K_a}(Ex)(b = x)$ | $\in \mu_n$ | } Assumption |
| 2. $\sim(Ex)\underline{K_a}(b = x)$ | $\in \mu_n$ | |
| 3. $(Ux)\underline{P_a}(b \neq x)$ | $\in \mu_n$ | 2 def.E def.F |
| 4. $(Ex)(b = x)$ | $\in \mu_n$ | 1 (C.K) |
| 5. $\underline{P_a}(b \neq b)$ | $\in \mu_n$ | 3, 4, (C.U _o) |
| 6. $(b \neq b)$ | $\in \mu_m$ | 5 (C.P*) |

which is contradictory.

(vii) The argument (A) on page 168 of the text is shown to be valid by the proof:-

- | | | | | |
|----|--|-------|---------|--------------------------|
| 1. | $\underline{K}_a(\underline{b} = \underline{c})$ | \in | μ_n | Premise |
| 2. | $(\underline{E}x)(\underline{x} = \underline{b})$ | \in | μ_n | Premise |
| 3. | $\sim(\underline{E}x)\underline{K}_a(\underline{x} = \underline{c})$ | \in | μ_n | Negated Conclusion |
| 4. | $(\underline{U}x)\underline{P}_a\sim(\underline{x} = \underline{c})$ | \in | μ_n | 3 def.E, def.P |
| 5. | $\underline{P}_a\sim(\underline{b} = \underline{c})$ | \in | μ_n | 2, 4 (C.U _o) |
| 6. | $\sim(\underline{b} = \underline{c})$ | \in | μ_m | 5 (C.P*) |
| 7. | $(\underline{b} = \underline{c})$ | \in | μ_m | 1, 5 (C.KK*)(C.K) |

which is contradictory.

(viii) A countermodel for argument (B) on page 169 of the text is:

$$\mu_n = \{ \underline{K}_a(\underline{b} = \underline{c}), (\underline{E}x)(\underline{x} = \underline{b}), \sim \underline{K}_a(\underline{E}x)(\underline{x} = \underline{c}), \underline{P}_a(\underline{U}x)\sim(\underline{x} = \underline{c}), (\underline{b} = \underline{c}) \}$$

$$\mu_m = \{ \underline{K}_a(\underline{b} = \underline{c}), (\underline{b} = \underline{c}), (\underline{U}x)\sim(\underline{x} = \underline{c}) \}$$

The system HK

In the text we indicated that a modified epistemic logic could be constructed.¹ We now set out that system - HK :

The primitive symbols of HK are those of HS4 together with one additional improper symbol : \underline{L}_P

The formation rules for HK are those of HS4 together with:

8. If \underline{A} is a wff and \underline{a} is any individual constant (personal) then $\underline{L}_P \underline{A}$ is a wff.

We define $\underline{L}_K \underline{A} =_{df} \sim \underline{L}_P \sim \underline{A}$

$\underline{K}_a \underline{A}$ is read as " \underline{a} knows that \underline{A} "

$\underline{L}_P \underline{A}$ is read as "It is logically possible, for all that \underline{a} knows, that \underline{A} "

1. This system is taken directly from the author's paper "Epistemic Logic, Language and Concepts" Logique et Analyse 64 (1973)

$\frac{P}{\underline{a}}$ is read as $\sim \underline{K}_{\underline{a}} \sim \underline{A}$, and $\frac{L}{\underline{K}} \underline{A}$ is read as $\sim \frac{L}{\underline{P}} \underline{a} \sim \underline{A}$.

The set of rules C^{HK} is made up as follows :

from Appendix II the rules $(C.\sim), (C.\sim\sim), (C.\supset), (C.\sim\supset), (C.E_o),$

$(C.U_o),$ and $(C.\text{self } \nrightarrow),$

and from Appendix III the rules $(C.K=), (C.=K)$ and $(C.=P).$

Also we construct the rules $(C.\frac{L}{\underline{P}}*), (C.\frac{L}{\underline{K}}), (C.\frac{L}{\underline{K}}\frac{L}{\underline{K}}*), (C.=\frac{L}{\underline{K}})$

and $(C.=\frac{L}{\underline{P}})$

from the rules $(C.P*), (C.K), (C.KK*),$

$(C.=K)$ and $(C.=P)$ respectively

by substituting $\frac{L}{\underline{K}}$ for \underline{K} and $\frac{L}{\underline{P}}$

for \underline{P} wherever \underline{K} and \underline{P} occur.

(In other words, we take HS4 and convert it into an epistemic logic for $\frac{L}{\underline{P}}$. What we now need are rules for \underline{K} , and rules to link the logics for $\frac{L}{\underline{P}}$ and \underline{K} together.)

The following rules are also added:

(C.P) If $\frac{P}{\underline{a}} \in \mu_n \in \Omega$ there is at least one member of Ω ,

such as μ_m , such that $\underline{a} \in \mu_m$ and μ_m is an ^oalternative to μ_n with respect to \underline{a} in Ω provided that μ_n is not an ^oalternative to any member of Ω with respect to \underline{a} ;

but if μ_n is an ^oalternative to some member of Ω with respect to \underline{a} then there is at least one member of Ω ,

such as μ_k , such that $\sim \underline{a} \in \mu_k$ and μ_k is an ^oalternative to μ_n with respect to \underline{a} in Ω .

(C.K~) If $\frac{K}{\underline{a}} \in \mu_n \in \Omega$ and μ_n is not an ^oalternative to any member of Ω with respect to \underline{a} and μ_m is an ^oalternative to μ_n with respect to \underline{a} in Ω then $\underline{a} \in \mu_m$;

but if μ_n is an ^oalternative to some member of Ω with respect to \underline{a} and μ_k is an ^oalternative to μ_n with

respect to \underline{a} in Ω then $\sim \underline{A} \in \mu_k$.

(C.K) (as in Appendix II)

(C.^LPP) If $\underline{L_{\underline{P}} \underline{A}} \in \mu_n \in \Omega$ then $\underline{P_{\underline{A}}} \in \mu_n$.

(C.K^LK) If $\underline{K_{\underline{A}}} \in \mu_n \in \Omega$ then $\underline{L_{\underline{K}} \underline{K_{\underline{A}}}} \in \mu_n$.

(The last two rules link the epistemic modalities)

It must be noted that in this epistemic logic it will be necessary, when showing that a formula is self-sustaining, to test its negation for membership of both ^oalternative sets and non-^oalternative sets.

For example, if we test the formula:

$$\underline{K_{\underline{A}}} \& \underline{K_{\underline{A}}} (\underline{A} \supset \underline{B}) \supset \underline{K_{\underline{A}}} \underline{B}$$

we first take an arbitrary set μ_n which is not an ^oalternative to any set:-

- | | | | |
|----|--|-------------|---------------------|
| 1. | $\underline{K_{\underline{A}}} \& \underline{K_{\underline{A}}} (\underline{A} \supset \underline{B})$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim \underline{K_{\underline{A}}} \underline{B}$ | $\in \mu_n$ | |
| 3. | $\underline{P_{\underline{A}}} \sim \underline{B}$ | $\in \mu_n$ | 2 def. _P |
| 4. | $\sim \underline{B}$ | $\in \mu_m$ | 3 (C.P) |
| 5. | \underline{A} | $\in \mu_m$ | 1 def.& (C.K~) |
| 6. | $(\underline{A} \supset \underline{B})$ | $\in \mu_m$ | 1 def.& (C.K~) |

which is contradictory;

but since the negation of the formula being tested might be satisfiable in an ^oalternative set we take an arbitrary set μ_n which is an ^oalternative to some set in :-

- | | | | |
|----|--|-------------|---------------------|
| 1. | $\underline{K_{\underline{A}}} \& \underline{K_{\underline{A}}} (\underline{A} \supset \underline{B})$ | $\in \mu_n$ | } Assumption |
| 2. | $\sim \underline{K_{\underline{A}}} \underline{B}$ | $\in \mu_n$ | |
| 3. | $\underline{P_{\underline{A}}} \sim \underline{B}$ | $\in \mu_n$ | 2 def. _P |
| 4. | $\sim \sim \underline{B}$ | $\in \mu_m$ | 3 (C.P) |
| 5. | \underline{B} | $\in \mu_m$ | 4 (C.~~) |
| 6. | $\sim \underline{A}$ | $\in \mu_m$ | 1 def.& (C.K~) |

7. $\sim(\underline{A} \supset \underline{B})$ $\in \mu_m$ 1 def.& (C.K \sim)
 8. \underline{A} $\in \mu_m$ 7 (C. \sim)

which is contradictory.

So the formula tested is self-sustaining in HK.

On the other hand, if \underline{T} is a tautology then $\underline{K_a T}$ will not be self-sustaining in HK. Although the negation of $\underline{K_a T}$ cannot be a member of a non- 0 alternative set, it can be a member of an 0 alternative set.

Assuming μ_n to be an 0 alternative to some member of Ω :-

1. $\sim \underline{K_a T}$ $\in \mu_n$ Assumption
 2. $\underline{P_a \sim T}$ $\in \mu_n$ 1 def. \underline{P}
 3. $\sim \sim \underline{T}$ $\in \mu_k$ 2 (C.P)
 4. \underline{T} $\in \mu_k$ 3 (C. $\sim \sim$)

which is not contradictory.

The system HK can be axiomatized as follows:

If A , B and C are wffs of HK, and X and Y are individual variables, and a , b and c are individual constants, and K is either \underline{K} or $\underline{L_K}$, then the axioms are

- (A1) $(A \supset (B \supset A))$
 (A2) $((A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C)))$
 (A3) $((\sim A \supset \sim B) \supset (B \supset A))$
 (A4) $(\underline{K_a} A \supset A)$
 (A5) $(\underline{K_a} (A \supset B) \supset (\underline{K_a} A \supset \underline{K_a} B))$
 (A6) $(\underline{L_K}_a A \supset \underline{L_K}_a \underline{L_K}_a A)$
 (A7) $(\underline{K_a} A \supset \underline{L_K}_a A)$
 (A8) $(\underline{K_a} A \supset \underline{L_K}_a \underline{K_a} A)$

- (A9) $((UX)A \supset ((EX)(X = a) \supset (A(a//X))))$
- (A10) $((UX)(A \supset B) \supset ((UX)A \supset (UX)B))$
- (A11) $(UX)(EY)(X = Y)$
- (A12) $(A \supset (UX)A)$
- (A13) $a = a$
- (A14) $(a = b \supset (B \supset (B(a/b))))$ provided B contains no modal operators.

Nothing else is an axiom.

The rules of inference are

$$\begin{array}{lcl}
 A, (A \supset B) & \rightarrow & B \\
 A & \rightarrow & L_{\underline{K}_a} A
 \end{array}$$

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